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(54) Title: METHOD AND SYSTEM OF COMPUTING AND RENDERING THE NATURE OF THE EXCITED ELECTRONIC STATES OF ATOMS AND ATOMIC IONS

(57) Abstract: A method and system of physically solving the charge, mass, and current density functions of excited-state atoms and atomic ions using Maxwell's equations and computing and rendering the nature of excited-state electrons using the solutions. The results can be displayed on visual or graphical media. The display can be static or dynamic such that electron spin and rotation motion can be displayed in an embodiment. The displayed information is useful to anticipate reactivity and physical properties. The insight into the nature of excited-state electrons can permit the solution and display of those of other atoms and atomic ions and provide utility to anticipate their reactivity and physical properties as well as spectral absorption and emission to lead to new optical materials and light sources.

METHOD AND SYSTEM OF COMPUTING AND RENDERING THE NATURE OF THE EXCITED ELECTRONIC STATES OF ATOMS AND ATOMIC IONS

I. INTRODUCTION

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1. Field of the Invention:

This invention relates to a method and system of physically solving the charge, mass, and current density functions of excited states of atoms and atomic ions and computing and rendering the nature of these species using the solutions. The results can

10 be displayed on visual or graphical media. The displayed information is useful to anticipate reactivity and physical properties. The insight into the nature of excited-state electrons can permit the solution and display of other excited-state atoms and ions and provide utility to anticipate their reactivity, physical properties, and spectral absorption and emission.

15 Rather than using postulated unverifiable theories that treat atomic particles as if they were not real, physical laws are now applied to atoms and ions. In an attempt to provide some physical insight into atomic problems and starting with the same essential physics as Bohr of the e^- moving in the Coulombic field of the proton with a true wave equation as opposed to the diffusion equation of Schrödinger, a classical approach is

20 explored which yields a model which is remarkably accurate and provides insight into physics on the atomic level. The proverbial view deeply seated in the wave-particle duality notion that there is no large-scale physical counterpart to the nature of the electron is shown not to be correct. Physical laws and intuition may be restored when dealing with the wave equation and quantum atomic problems.

25 Specifically, a theory of classical quantum mechanics (CQM) was derived from first principles as reported previously [reference Nos. 1-7] that successfully applies physical laws to the solution of atomic problems that has its basis in a breakthrough in the understanding of the stability of the bound electron to radiation. Rather than using the postulated Schrödinger boundary condition: "$\Psi \to 0$ as $r \to \infty$", which leads to a

30 purely mathematical model of the electron, the constraint is based on experimental observation. Using Maxwell's equations, *the classical wave equation is solved with the constraint that the bound $n = 1$-state electron cannot radiate energy*. Although it is well known that an accelerated *point* particle radiates, an *extended distribution* modeled as a superposition of accelerating charges does not have to radiate. A simple invariant

35 physical model arises naturally wherein the predicted results are extremely

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straightforward and internally consistent requiring minimal math as in the case of the most famous equations of Newton, Maxwell, Einstein, de Broglie, and Planck on which the model is based. No new physics is needed; only the known physical laws based on direct observation are used. The solution of the excited states of one-electron atoms is

5   given in R. Mills, *The Grand Unified Theory of Classical Quantum Mechanics*, January 2005 Edition, BlackLight Power, Inc., Cranbury, New Jersey, (" '05 Mills GUT") which is herein incorporated by reference. This Invention further comprises the accurate solution of the helium-atom excited states which provides a physical algorithm to solve the excited states of other multi-electron atoms.

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2. Background of the Invention

2A. Classical Quantum Theory of the Atom Based on Maxwell's Equations
The old view that the electron is a zero or one-dimensional point in an all-space

15   probability wave function $\Psi(x)$ is not taken for granted. The theory of classical quantum mechanics (CQM), derived from first principles, must successfully and consistently apply physical laws on all scales [1-7]. Stability to radiation was ignored by all past atomic models. Historically, the point at which QM broke with classical laws can be traced to the issue of nonradiation of the one electron atom. Bohr just postulated

20   orbits stable to radiation with the further postulate that the bound electron of the hydrogen atom does not obey Maxwell's equations—rather it obeys different physics [1-10]. Later physics was replaced by "pure mathematics" based on the notion of the inexplicable wave-particle duality nature of electrons which lead to the Schrödinger equation wherein the consequences of radiation predicted by Maxwell's equations were

25   ignored. Ironically, Bohr, Schrödinger, and Dirac used the Coulomb potential, and Dirac used the vector potential of Maxwell's equations. But, all ignored electrodynamics and the corresponding radiative consequences. Dirac originally attempted to solve the bound electron physically with stability with respect to radiation according to Maxwell's equations with the further constraints that it was relativistically invariant and gave rise to

30   electron spin [11]. He and many founders of QM such as Sommerfeld, Bohm, and Weinstein wrongly pursued a planetary model, were unsuccessful, and resorted to the current mathematical-probability-wave model that has many problems [10, 11-14]. Consequently, Feynman for example, attempted to use first principles including Maxwell's equations to discover new physics to replace quantum mechanics [15].

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Physical laws may indeed be the root of the observations thought to be "purely quantum mechanical", and it was a mistake to make the assumption that Maxwell's electrodynamic equations must be rejected at the atomic level. Thus, in the present approach, the classical wave equation is solved with the constraint that a bound $n = 1$ -

5   state electron cannot radiate energy.

Herein, derivations consider the electrodynamic effects of moving charges as well as the Coulomb potential, and the search is for a solution representative of the electron wherein there is acceleration of charge motion without radiation. The mathematical formulation for zero radiation based on Maxwell's equations follows from

10   a derivation by Haus [16]. The function that describes the motion of the electron must not possess spacetime Fourier components that are synchronous with waves traveling at the speed of light. Similarly, nonradiation is demonstrated based on the electron's electromagnetic fields and the Poynting power vector.

It was shown previously [1-7] that CQM gives closed form solutions for the atom

15   including the stability of the $n = 1$ state and the instability of the excited states, the equation of the photon and electron in excited states, the equation of the free electron, and photon which predict the wave particle duality behavior of particles and light. The current and charge density functions of the electron may be directly physically interpreted. For example, spin angular momentum results from the motion of negatively

20   charged mass moving systematically, and the equation for angular momentum, $\mathbf{r} \times \mathbf{p}$, can be applied directly to the wave function (a current density function) that describes the electron. The magnetic moment of a Bohr magneton, Stern Gerlach experiment, g factor, Lamb shift, resonant line width and shape, selection rules, correspondence principle, wave particle duality, excited states, reduced mass, rotational energies, and

25   momenta, orbital and spin splitting, spin-orbital coupling, Knight shift, and spin-nuclear coupling, and elastic electron scattering from helium atoms, are derived in closed-form equations based on Maxwell's equations. The calculations agree with experimental observations.

The Schrödinger equation gives a vague and fluid model of the electron.

30   Schrödinger interpreted $e\Psi * (x)\Psi(x)$ as the charge-density or the amount of charge between x and $x + dx$ ($\Psi *$ is the complex conjugate of Ψ). Presumably, then, he pictured the electron to be spread over large regions of space. After Schrödinger's interpretation, Max Born, who was working with scattering theory, found that this

interpretation led to inconsistencies, and he replaced the Schrödinger interpretation with
the probability of finding the electron between x and $x + dx$ as

$$\int \Psi(x)\Psi*(x)\,dx \qquad\qquad (1)$$

Born's interpretation is generally accepted. Nonetheless, interpretation of the wave

5   function is a never-ending source of confusion and conflict. Many scientists have solved
this problem by conveniently adopting the Schrödinger interpretation for some problems
and the Born interpretation for others. This duality allows the electron to be everywhere
at one time—yet have no volume. Alternatively, the electron can be viewed as a discrete
particle that moves here and there (from $r = 0$ to $r = \infty$), and $\Psi\Psi*$ gives the time

10   average of this motion.

In contrast to the failure of the Bohr theory and the nonphysical, adjustable-
parameter approach of quantum mechanics, multielectron atoms [1, 5] and the nature of
the chemical bond [1, 4] are given by exact closed-form solutions containing
fundamental constants only. Using the nonradiative wave equation solutions that

15   describe the bound electron having conserved momentum and energy, the radii are
determined from the force balance of the electric, magnetic, and centrifugal forces that
corresponds to the minimum of energy of the system. The ionization energies are then
given by the electric and magnetic energies at these radii. The spreadsheets to calculate
the energies from exact solutions of one through twenty-electron atoms are given in '05

20   Mills GUT [1] and are available from the internet [17]. For 400 atoms and ions the
agreement between the predicted and experimental results is remarkable.

The background theory of classical quantum mechanics (CQM) for the physical
solutions of atoms and atomic ions is disclosed in R. Mills, *The Grand Unified Theory of
Classical Quantum Mechanics*, January 2000 Edition, BlackLight Power, Inc., Cranbury,

25   New Jersey, (" '00 Mills GUT"), provided by BlackLight Power, Inc., 493 Old Trenton
Road, Cranbury, NJ, 08512; R. Mills, *The Grand Unified Theory of Classical Quantum
Mechanics*, September 2001 Edition, BlackLight Power, Inc., Cranbury, New Jersey,
Distributed by Amazon.com (" '01 Mills GUT"), provided by BlackLight Power, Inc.,
493 Old Trenton Road, Cranbury, NJ, 08512; R. Mills, *The Grand Unified Theory of

30   Classical Quantum Mechanics*, July 2004 Edition, BlackLight Power, Inc., Cranbury,
New Jersey, (" '04 Mills GUT"), provided by BlackLight Power, Inc., 493 Old Trenton
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Mechanics, January 2005 Edition, BlackLight Power, Inc., Cranbury, New Jersey, (" '05

Mills GUT"), provided by BlackLight Power, Inc., 493 Old Trenton Road, Cranbury, NJ,

08512 (posted at www.blacklightpower.com); in prior PCT applications

PCT/US02/35872; PCT/US02/06945; PCT/US02/06955; PCT/US01/09055; PCT/US01/

5   25954; PCT/US00/20820; PCT/US00/20819; PCT/US00/09055; PCT/US99/17171;

PCT/US99/17129; PCT/US 98/22822; PCT/US98/14029; PCT/US96/07949;

PCT/US94/02219; PCT/US91/08496; PCT/US90/01998; and PCT/US89/05037 and U.S.

Patent No. 6,024,935; the entire disclosures of which are all incorporated herein by

reference; (hereinafter "Mills Prior Publications").

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II. SUMMARY OF THE INVENTION

An object of the present invention is to solve the charge (mass) and current-

density functions of excited-state atoms and atomic ions from first principles. In an

embodiment, the solution for the excited and non-excited state is derived from Maxwell's

15   equations invoking the constraint that the bound electron before excitation does not

radiate even though it undergoes acceleration.

Another objective of the present invention is to generate a readout, display, or

image of the solutions so that the nature of excited-state atoms and atomic ions can be

better understood and potentially applied to predict reactivity and physical and optical

20   properties.

Another objective of the present invention is to apply the methods and systems of

solving the nature of excited-state electrons and their rendering to numerical or graphical

form to all atoms and atomic ions.

Bound electrons are described by a charge-density (mass-density) function which

25   is the product of a radial delta function ($f(r) = \delta(r - r_n)$), two angular functions

(spherical harmonic functions), and a time harmonic function. Thus, a bound electron is

a dynamic "bubble-like" charge-density function. The two-dimensional spherical

surface called an electron orbitsphere shown in FIGURE 1 can exist in a bound state at

only specified distances from the nucleus. More explicitly, the orbitsphere comprises a

30   two-dimensional spherical shell of moving charge. The current pattern of the

orbitsphere that gives rise to the phenomenon corresponding to the spin quantum number

comprises an infinite series of correlated orthogonal great circle current loops. As given

in the Orbitsphere Equation of Motion for $\ell = 0$ section of '05 Mills GUT [1], the current

pattern (shown in FIGURE 2) is generated over the surface by two orthogonal sets of an infinite series of nested rotations of two orthogonal great circle current loops where the coordinate axes rotate with the two orthogonal great circles. Each infinitesimal rotation of the infinite series is about the new x-axis and new y-axis which results from the

5   preceding such rotation. For each of the two sets of nested rotations, the angular sum of the rotations about each rotating x-axis and y-axis totals $\sqrt{2}\,\pi$ radians. The spin function of the electron corresponds to the nonradiative $n = 1$, $\ell = 0$ state which is well known as an s state or orbital. (See FIGURE 1 for the charge function and FIGURE 2 for the current function.) In cases of orbitals of excited states with the ℓ quantum number not

10   equal to zero and which are not constant as given by Eq. (1.64) of Ref. [1], the constant spin function is modulated by a time and spherical harmonic function as given by Eq. (1.65) of Ref. [1] and shown in FIGURE 3. The modulation or traveling charge-density wave corresponds to an orbital angular momentum in addition to a spin angular momentum. These states are typically referred to as p, d, f, etc. orbitals.

15   Each orbitsphere is a spherical shell of negative charge (total charge $= -e$) of zero thickness at a distance r_n from the nucleus (charge $= +Ze$). It is well known that the field of a spherical shell of charge is zero inside the shell and that of a point charge at the origin outside the shell [1] (See FIGURE 1.12 of Ref. [1]). The field of each electron can be treated as that corresponding to a $-e$ charge at the origin with $\mathbf{E} = \dfrac{-e}{4\pi\varepsilon_o r^2}$ for

20   $r > r_n$ and $\mathbf{E} = 0$ for $r < r_n$ where r_n is the radius of the electron orbitsphere. Thus, as shown in the Two-Electron Atom section of '05 Mills GUT [1], the central electric fields due to the helium nucleus are $\mathbf{E} = \dfrac{2e}{4\pi\varepsilon_o r^2}$ and $\mathbf{E} = \dfrac{e}{4\pi\varepsilon_o r^2}$ for $r < r_1$ and $r_1 < r < r_2$, respectively. In the ground state of the helium atom, both electrons are at $r_1 = r_2 = 0.567 a_o$. When a photon is absorbed, one of the initially indistinguishable

25   electrons called electron 1 moves to a smaller radius, and the other called electron 2 moves to a greater radius. In the limiting case of the absorption of an ionizing photon, electron 1 moves to the radius of the helium ion, $r_1 = 0.5 a_o$, and electron 2 moves to a continuum radius, $r_2 = \infty$. When a photon is absorbed by the ground state helium atom it generates an effective charge, $Z_{P-\mathit{eff}}$, within the second orbitsphere such that the

30   electrons move in opposite radial directions while conserving energy and angular momentum. We can determine $Z_{P-\mathit{eff}}$ of the "trapped photon" electric field by requiring

that the resonance condition is met for photons of discrete energy, frequency, and wavelength for electron excitation in an electromagnetic potential energy well.

In contrast to the shortcomings of quantum mechanics, with classical quantum mechanics (CQM), all excited states of the helium atom can be exactly solved in closed

5   form. Photon absorption occurs by an excitation of a Maxwellian multipole cavity mode wherein the excitation is quantized according to the quantized energy and angular momentum of the photon given by $\hbar\omega$ and $\hbar$, respectively. The photon quantization causes the central electric-field corresponding the superimposed fields of the nucleus, electron 1, and the photon to be quantized and of magnitude of a reciprocal integer times

10   that of the proton. This field and the phase-matched angular dependence of the trapped photon and excited-state electron as well as the spin orientation of the excited-state electron determine the central forces. The radii of electron 2 are determined from the force balance of the electric, magnetic, and centrifugal forces that corresponds to the minimum of energy of the system. Since the magnetic energies are relatively

15   insignificant, in one embodiment, the excited state energies are then given by one physical term in each case, the Coulombic energy at the calculated radius. In additional embodiments, additional small terms may refine the solutions. Given the typical average relative difference is about 5 significant figures which is within the error of the experimental data, this result is remarkable and strongly confirms that the physical CQM

20   solution of helium is correct.

The presented exact physical solutions for the excited states of the helium atom can be applied to other atoms and ions to solve for their excited states. These solution can be used to predict the properties of elements and ions and engineer compositions of matter in a manner which is not possible using quantum mechanics. It also for the

25   prediction of the spectral absorption and emission. This in tern can be used to develop new light filters or absorbers as well as new light sources such as lasers, lamps, and spectral standards.

In an embodiment., the physical, Maxwellian solutions for the dimensions and energies of excited-state atom and atomic ions are processed with a processing means to

30   produce an output. Embodiments of the system for performing computing and rendering of the nature of the excited-state atomic and atomic-ionic electrons using the physical solutions may comprise a general purpose computer. Such a general purpose computer may have any number of basic configurations. For example, such a general purpose

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computer may comprise a central processing unit (CPU), one or more specialized processors, system memory, a mass storage device such as a magnetic disk, an optical disk, or other storage device, an input means such as a keyboard or mouse, a display device, and a printer or other output device. A system implementing the present

5   invention can also comprise a special purpose computer or other hardware system and all should be included within its scope.

III. BRIEF DESCRIPTION OF THE DRAWINGS

FIGURE 1 shows the orbitsphere in accordance with the present invention that is

10   a two dimensional spherical shell of zero thickness with the Bohr radius of the hydrogen atom, $r = a_H$.

FIGURE 2 shows the current pattern of the orbitsphere in accordance with the present invention from the perspective of looking along the z-axis. The current and charge density are confined to two dimensions at $r_n = nr_1$. The corresponding charge

15   density function is uniform.

FIGURE 3 shows that the orbital function modulates the constant (spin) function (shown for t = 0; three-dimensional view).

FIGURE 4 shows the normalized radius as a function of the velocity due to relativistic contraction.

20   FIGURE 5 shows the magnetic field of an electron orbitsphere (z-axis defined as the vertical axis).

FIGURE 6 shows a plot of the predicted and experimental energies of levels assigned by NIST, and

FIGURE 7 shows a computer rendering of the helium atom in the n=2, $\ell = 1$

25   excited state according to the present Invention.

IV. DETAILED DESCRIPTION OF THE INVENTION

The following preferred embodiments of the invention disclose numerous

30   calculations which are merely intended as illustrative examples. Based on the detailed written description, one skilled in the art would easily be able to practice this Invention within other like calculations to produce the desired result without undue effort.

9

1. ONE-ELECTRON ATOMS

1. ONE-ELECTRON ATOMS

One-electron atoms include the hydrogen atom, He^+, Li^{2+}, Be^{3+}, and so on. The

5 mass-energy and angular momentum of the electron are constant; this requires that the

equation of motion of the electron be temporally and spatially harmonic. Thus, the

classical wave equation applies and

$$\left[\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right] \rho(r,\theta,\phi,t) = 0 \tag{2}$$

where $\rho(r,\theta,\phi,t)$ is the time dependent charge density function of the electron in time

10 and space. In general, the wave equation has an infinite number of solutions. To arrive

at the solution which represents the electron, a suitable boundary condition must be

imposed. It is well known from experiments that each single atomic electron of a given

isotope radiates to the same stable state. Thus, the physical boundary condition of

nonradiation of the bound electron was imposed on the solution of the wave equation for

15 the time dependent charge density function of the electron [1-3, 5]. The condition for

radiation by a moving point charge given by Haus [16] is that its spacetime Fourier

transform does possess components that are synchronous with waves traveling at the

speed of light. Conversely, it is proposed that the condition for nonradiation by an

ensemble of moving point charges that comprises a current density function is

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For non-radiative states, the current-density function must NOT possess spacetime
Fourier components that are synchronous with waves traveling at the speed of light.

The time, radial, and angular solutions of the wave equation are separable. The motion

25 is time harmonic with frequency ω_n. A constant angular function is a solution to the

wave equation. Solutions of the Schrödinger wave equation comprising a radial function

radiate according to Maxwell's equation as shown previously by application of Haus'

condition [1]. In fact, it was found that any function which permitted radial motion gave

rise to radiation. A radial function which does satisfy the boundary condition is a radial

30 delta function

$$f(r) = \frac{1}{r^2} \delta(r - r_n) \tag{3}$$

10

This function defines a constant charge density on a spherical shell where $r_n = nr_1$

wherein n is an integer in an excited state, and Eq. (2) becomes the two-dimensional

wave equation plus time with separable time and angular functions. Given time

harmonic motion and a radial delta function, the relationship between an allowed radius

5   and the electron wavelength is given by

$$2\pi r_n = \lambda_n \tag{4}$$

where the integer subscript n here and in Eq. (3) is determined during photon absorption

as given in the Excited States of the One-Electron Atom (Quantization) section of Ref.

[1]. Using the observed de Broglie relationship for the electron mass where the

10   coordinates are spherical,

$$\lambda_n = \frac{h}{p_n} = \frac{h}{m_e v_n} \tag{5}$$

and the magnitude of the velocity for *every* point on the orbitsphere is

$$v_n = \frac{\hbar}{m_e r_n} \tag{6}$$

The sum of the $|\mathbf{L}_i|$, the magnitude of the angular momentum of each infinitesimal point

15   of the orbitsphere of mass m_i, must be constant. The constant is $\hbar$.

$$\sum |\mathbf{L}_i| = \sum |\mathbf{r} \times m_i \mathbf{v}| = m_e r_n \frac{\hbar}{m_e r_n} = \hbar \tag{7}$$

Thus, an electron is a spinning, two-dimensional spherical surface (zero thickness),

called an *electron orbitsphere* shown in FIGURE 1, that can exist in a bound state at

only specified distances from the nucleus determined by an energy minimum. The

20   corresponding current function shown in FIGURE 2 which gives rise to the phenomenon

of *spin* is derived in the Spin Function section. (See the Orbitsphere Equation of Motion

for $\ell = 0$ of Ref. [1] at Chp. 1.)

Nonconstant functions are also solutions for the angular functions. To be a

harmonic solution of the wave equation in spherical coordinates, these angular functions

25   must be spherical harmonic functions [18]. A zero of the spacetime Fourier transform of

the product function of two spherical harmonic angular functions, a time harmonic

function, and an unknown radial function is sought. The solution for the radial function

which satisfies the boundary condition is also a delta function given by Eq. (3). Thus,

bound electrons are described by a charge-density (mass-density) function which is the

30   product of a radial delta function, two angular functions (spherical harmonic functions),

and a time harmonic function.

$$\rho(r,\theta,\phi,t) = f(r)A(\theta,\phi,t) = \frac{1}{r^2}\delta(r - r_n)A(\theta,\phi,t); \qquad A(\theta,\phi,t) = Y(\theta,\phi)k(t) \quad (8)$$

In these cases, the spherical harmonic functions correspond to a traveling charge density

wave confined to the spherical shell which gives rise to the phenomenon of orbital

5   angular momentum. The orbital functions which modulate the constant "spin" function

shown graphically in FIGURE 3 are given in the Sec. 1.B.

1.A. SPIN FUNCTION

The orbitsphere spin function comprises a constant charge (current) density

10   function with moving charge confined to a two-dimensional spherical shell. The

magnetostatic current pattern of the orbitsphere spin function comprises an infinite series

of correlated orthogonal great circle current loops wherein each point charge (current)

density element moves time harmonically with constant angular velocity

$$\omega_n = \frac{\hbar}{m_e r_n^2} \tag{9}$$

15   The uniform current density function $Y_0^0(\phi,\theta)$, the orbitsphere equation of

motion of the electron (Eqs. (14-15)), corresponding to the constant charge function of

the orbitsphere that gives rise to the spin of the electron is generated from a basis set

current-vector field defined as the orbitsphere current-vector field ("orbitsphere-cvf").

This in turn is generated over the surface by two complementary steps of an infinite

20   series of nested rotations of two orthogonal great circle current loops where the

coordinate axes rotate with the two orthogonal great circles that serve as a basis set. The

algorithm to generate the current density function rotates the great circles and the

corresponding x'y'z' coordinates relative to the xyz frame. Each infinitesimal rotation of

the infinite series is about the new i'-axis and new j'-axis which results from the

25   preceding such rotation. Each element of the current density function is obtained with

each conjugate set of rotations. In Appendix III of Ref. [1], the *continuous* uniform

electron current density function $Y_0^0(\phi,\theta)$ having the same angular momentum

components as that of the orbitsphere-cvf is then exactly generated from this orbitsphere-

cvf as a basis element by a convolution operator comprising an autocorrelation-type

30   function.

For Step One, the current density elements move counter clockwise on the great

circle in the y'z'-plane and move clockwise on the great circle in the x'z'-plane. The great circles are rotated by an infinitesimal angle $\pm\Delta\alpha_r$ (a positive rotation around the x'-axis or a negative rotation about the z'-axis for Steps One and Two, respectively) and then by $\pm\Delta\alpha_{j'}$ (a positive rotation around the new y'-axis or a positive rotation about the new x'-axis for Steps One and Two, respectively). The coordinates of each point on each rotated great circle (x',y',z') is expressed in terms of the first (x,y,z) coordinates by the following transforms where clockwise rotations and motions are defined as positive looking along the corresponding axis:

Step One

$$
\begin{bmatrix} x \\ y \\ z \end{bmatrix} =
\begin{bmatrix} \cos(\Delta\alpha_y) & 0 & -\sin(\Delta\alpha_y) \\ 0 & 1 & 0 \\ \sin(\Delta\alpha_y) & 0 & \cos(\Delta\alpha_y) \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Delta\alpha_x) & \sin(\Delta\alpha_x) \\ 0 & -\sin(\Delta\alpha_x) & \cos(\Delta\alpha_x) \end{bmatrix}
\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}
$$

$$
\begin{bmatrix} x \\ y \\ z \end{bmatrix} =
\begin{bmatrix} \cos(\Delta\alpha_y) & \sin(\Delta\alpha_y)\sin(\Delta\alpha_x) & -\sin(\Delta\alpha_y)\cos(\Delta\alpha_x) \\ 0 & \cos(\Delta\alpha_x) & \sin(\Delta\alpha_x) \\ \sin(\Delta\alpha_y) & -\cos(\Delta\alpha_y)\sin(\Delta\alpha_x) & \cos(\Delta\alpha_y)\cos(\Delta\alpha_x) \end{bmatrix}
\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}
$$

$$(10)$$

Step Two

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\Delta\alpha_x) & \sin(\Delta\alpha_x) \\ 0 & -\sin(\Delta\alpha_x) & \cos(\Delta\alpha_x) \end{bmatrix} \begin{bmatrix} \cos(\Delta\alpha_z) & \sin(\Delta\alpha_z) & 0 \\ -\sin(\Delta\alpha_z) & \cos(\Delta\alpha_z) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos(\Delta\alpha_z) & \sin(\Delta\alpha_z) & 0 \\ -\cos(\Delta\alpha_x)\sin(\Delta\alpha_z) & \cos(\Delta\alpha_x)\cos(\Delta\alpha_z) & \sin(\Delta\alpha_x) \\ \sin(\Delta\alpha_x)\sin(\Delta\alpha_z) & -\sin(\Delta\alpha_x)\cos(\Delta\alpha_z) & \cos(\Delta\alpha_x) \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

(11)

where the angular sum is $\lim\limits_{\Delta\alpha \to 0} \sum\limits_{n=1}^{\frac{\frac{\sqrt{2}}{2}\pi}{|\Delta\alpha_{i',j'}|}} |\Delta\alpha_{i',j'}| = \frac{\sqrt{2}}{2}\pi$.

5   The orbitsphere-cvf is given by n reiterations of Eqs. (10) and (11) for each point on each of the two orthogonal great circles during each of Steps One and Two. The output given by the non-primed coordinates is the input of the next iteration corresponding to each successive nested rotation by the infinitesimal angle $\pm\Delta\alpha_{i'}$ or $\pm\Delta\alpha_{j'}$ where the magnitude of the angular sum of the n rotations about each of the i'-

10   axis and the j'-axis is $\frac{\sqrt{2}}{2}\pi$. Half of the orbitsphere-cvf is generated during each of Steps One and Two.

        Following Step Two, in order to match the boundary condition that the magnitude of the velocity at any given point on the surface is given by Eq. (6), the output half of the orbitsphere-cvf is rotated clockwise by an angle of $\frac{\pi}{4}$ about the z-axis.

15   Using Eq. (11) with $\Delta\alpha_{z'} = \frac{\pi}{4}$ and $\Delta\alpha_{x'} = 0$ gives the rotation. Then, the one half of the orbitsphere-cvf generated from Step One is superimposed with the complementary half obtained from Step Two following its rotation about the z-axis of $\frac{\pi}{4}$ to give the basis function to generate $Y_0^0(\phi,\theta)$, the orbitsphere equation of motion of the electron.

        The current pattern of the orbitsphere-cvf generated by the nested rotations of the

20   orthogonal great circle current loops is a continuous and total coverage of the spherical

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surface, but it is shown as a visual representation using 6 degree increments of the infinitesimal angular variable $\pm\Delta\alpha_{i'}$ and $\pm\Delta\alpha_{j'}$ of Eqs. (10) and (11) from the perspective of the z-axis in FIGURE 2. In each case, the complete orbitsphere-cvf current pattern corresponds all the orthogonal-great-circle elements which are generated

5   by the rotation of the basis-set according to Eqs. (10) and (11) where $\pm\Delta\alpha_{i'}$ and $\pm\Delta\alpha_{j'}$ approach zero and the summation of the infinitesimal angular rotations of $\pm\Delta\alpha_{i'}$ and

$\pm\Delta\alpha_{j'}$ about the successive i'-axes and j'-axes is $\dfrac{\sqrt{2}}{2}\pi$ for each Step. The current

pattern gives rise to the phenomenon corresponding to the spin quantum number. The details of the derivation of the spin function are given in Ref. [3] and Chp. 1 of Ref. [1].

10   The resultant angular momentum projections of $\mathbf{L}_{xy} = \dfrac{\hbar}{4}$ and $\mathbf{L}_z = \dfrac{\hbar}{2}$ meet the

boundary condition for the unique current having an angular velocity magnitude at each point on the surface given by Eq. (6) and give rise to the Stern Gerlach experiment as shown in Ref. [1]. The further constraint that the current density is uniform such that the charge density is uniform, corresponding to an equipotential, minimum energy surface is

15   satisfied by using the orbitsphere-cvf as a basis element to generate $Y_0^0(\phi,\theta)$ using a convolution operator comprising an autocorrelation-type function as given in Appendix III of Ref. [1]. The operator comprises the convolution of each great circle current loop of the orbitsphere-cvf designated as the primary orbitsphere-cvf with a second orbitsphere-cvf designated as the secondary orbitsphere-cvf wherein the convolved

20   secondary elements are matched for orientation, angular momentum, and phase to those of the primary. The resulting exact uniform current distribution obtained from the convolution has the same angular momentum distribution, resultant, $\mathbf{L_R}$, and

components of $\mathbf{L}_{xy} = \dfrac{\hbar}{4}$ and $\mathbf{L}_z = \dfrac{\hbar}{2}$ as those of the orbitsphere-cvf used as a primary

basis element.

25

1.B. ANGULAR FUNCTIONS

The time, radial, and angular solutions of the wave equation are separable. Also based on the radial solution, the angular charge and current-density functions of the electron, $A(\theta, \phi, t)$, must be a solution of the wave equation in two dimensions (plus

30   time),

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]A(\theta,\phi,t) = 0 \qquad (12)$$

where $\rho(r,\theta,\phi,t) = f(r)A(\theta,\phi,t) = \frac{1}{r^2}\delta(r - r_n)A(\theta,\phi,t)$ and $A(\theta,\phi,t) = Y(\theta,\phi)k(t)$

$$\left[\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)_{r,\phi} + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)_{r,\theta} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]A(\theta,\phi,t) = 0 \qquad (13)$$

where v is the linear velocity of the electron. The charge-density functions including the time-function factor are

$\ell = 0$

$$\rho(r,\theta,\phi,t) = \frac{e}{8\pi r^2}[\delta(r - r_n)]\left[Y_0^0(\theta,\phi) + Y_\ell^m(\theta,\phi)\right] \qquad (14)$$

$\ell \neq 0$

$$\rho(r,\theta,\phi,t) = \frac{e}{4\pi r^2}[\delta(r - r_n)]\left[Y_0^0(\theta,\phi) + \mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\}\right] \qquad (15)$$

where $Y_\ell^m(\theta,\phi)$ are the spherical harmonic functions that spin about the z-axis with angular frequency ω_n with $Y_0^0(\theta,\phi)$ the constant function.

$\mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\} = P_\ell^m(\cos\theta)\cos(m\phi + \omega_n' t)$ where to keep the form of the spherical harmonic as a traveling wave about the z-axis, $\omega_n' = m\omega_n$.

1.C. ACCELERATION WITHOUT RADIATION

1.C.a. Special Relativistic Correction to the Electron Radius

The relationship between the electron wavelength and its radius is given by Eq. (4) where λ is the de Broglie wavelength. For each current density element of the spin function, the distance along each great circle in the direction of instantaneous motion undergoes length contraction and time dilation. Using a phase matching condition, the wavelengths of the electron and laboratory inertial frames are equated, and the corrected radius is given by

$$r_n = r_n'\left[\sqrt{1 - \left(\frac{v}{c}\right)^2}\sin\left[\frac{\pi}{2}\left(1 - \left(\frac{v}{c}\right)^2\right)^{3/2}\right] + \frac{1}{2\pi}\cos\left[\frac{\pi}{2}\left(1 - \left(\frac{v}{c}\right)^2\right)^{3/2}\right]\right] \qquad (16)$$

where the electron velocity is given by Eq. (6). (See Ref. [1] Chp. 1, Special Relativistic Correction to the Ionization Energies section). $\frac{e}{m_e}$ of the electron, the electron angular

16

momentum of $\hbar$, and μ_B are invariant, but the mass and charge densities increase in the laboratory frame due to the relativistically contracted electron radius. As $v \to c$,

$r/r' \to \dfrac{1}{2\pi}$ and $r = \lambda$ as shown in FIGURE 4.

5   1.C.b. Nonradiation Based on the Spacetime Fourier Transform of the Electron Current
         The Fourier transform of the electron charge density function given by Eq. (8) is a solution of the three-dimensional wave equation in frequency space ($\mathbf{k}, \omega$ space) as given in Chp 1, Spacetime Fourier Transform of the Electron Function section of Ref. [1]. Then, the corresponding Fourier transform of the current density function

10   $K(s, \Theta, \Phi, \omega)$ is given by multiplying by the constant angular frequency.

$$K(s,\Theta,\Phi,\omega) = 4\pi\omega_n \frac{\sin(2s_n r_n)}{2s_n r_n} \otimes 2\pi \sum_{\upsilon=1}^{\infty} \frac{(-1)^{\upsilon-1}(\pi\sin\Theta)^{2(\upsilon-1)}}{(\upsilon-1)!(\upsilon-1)!} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\upsilon+\frac{1}{2}\right)}{(\pi\cos\Theta)^{2\upsilon+1}2^{\upsilon+1}} \frac{2\upsilon!}{(\upsilon-1)!} s^{-2\upsilon}$$

$$\otimes 2\pi \sum_{\upsilon=1}^{\infty} \frac{(-1)^{\upsilon-1}(\pi\sin\Phi)^{2(\upsilon-1)}}{(\upsilon-1)!(\upsilon-1)!} \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\upsilon+\frac{1}{2}\right)}{(\pi\cos\Phi)^{2\upsilon+1}2^{\upsilon+1}} \frac{2\upsilon!}{(\upsilon-1)!} s^{-2\upsilon} \frac{1}{4\pi}[\delta(\omega-\omega_n)+\delta(\omega+\omega_n)]$$

(17)

15   $\mathbf{s}_n \bullet \mathbf{v}_n = \mathbf{s}_n \bullet \mathbf{c} = \omega_n$ implies $r_n = \lambda_n$ which is given by Eq. (16) in the case that k is the lightlike k^0. In this case, Eq. (17) vanishes. Consequently, spacetime harmonics of

$\dfrac{\omega_n}{c} = k$ or $\dfrac{\omega_n}{c}\sqrt{\dfrac{\varepsilon}{\varepsilon_o}} = k$ for which the Fourier transform of the current-density function is

nonzero do not exist. Radiation due to charge motion does not occur in any medium when this boundary condition is met. Nonradiation is also determined directly from the

20   fields based on Maxwell's equations as given in Sec. 1.C.c.

1.C.c Nonradiation Based on the Electron Electromagnetic Fields and the Poynting Power Vector
         A point charge undergoing periodic motion accelerates and as a consequence

25   radiates according to the Larmor formula:

$$P = \frac{1}{4\pi\varepsilon_0}\frac{2e^2}{3c^3}a^2$$

(18)

where e is the charge, a is its acceleration, ε_0 is the permittivity of free space, and c is the speed of light. Although an accelerated *point* particle radiates, an *extended distribution* modeled as a superposition of accelerating charges does not have to radiate

30   [11, 16, 19-21]. In Ref. [3] and Appendix I, Chp. 1 of Ref. [1], the electromagnetic far

field is determined from the current distribution in order to obtain the condition, if it exists, that the electron current distribution must satisfy such that the electron does not radiate. The current follows from Eqs. (14-15). The currents corresponding to Eq. (14) and first term of Eq. (15) are static. Thus, they are trivially nonradiative. The current due to the time dependent term of Eq. (15) corresponding to p, d, f, etc. orbitals is

$$
\begin{aligned}
\mathbf{J} &= \frac{\omega_n}{2\pi}\frac{e}{4\pi r_n^2}N[\delta(r-r_n)]\operatorname{Re}\left\{Y_\ell^m(\theta,\phi)\right\}[\mathbf{u}(t)\times\mathbf{r}] \\
&= \frac{\omega_n}{2\pi}\frac{e}{4\pi r_n^2}N'[\delta(r-r_n)]\left(P_\ell^m(\cos\theta)\cos(m\phi+\omega_n't)\right)[\mathbf{u}\times\mathbf{r}] \qquad (19) \\
&= \frac{\omega_n}{2\pi}\frac{e}{4\pi r_n^2}N'[\delta(r-r_n)]\left(P_\ell^m(\cos\theta)\cos(m\phi+\omega_n't)\right)\sin\theta\hat{\phi}
\end{aligned}
$$

where to keep the form of the spherical harmonic as a traveling wave about the z-axis, $\omega_n' = m\omega_n$ and N and N' are normalization constants. The vectors are defined as

$$
\hat{\phi} = \frac{\hat{u}\times\hat{r}}{|\hat{u}\times\hat{r}|} = \frac{\hat{u}\times\hat{r}}{\sin\theta}; \qquad \hat{u} = \hat{z} = orbital\ axis \qquad (20)
$$

$$
\hat{\theta} = \hat{\phi}\times\hat{r} \qquad (21)
$$

"∧" denotes the unit vectors $\hat{u}\equiv\dfrac{\mathbf{u}}{|\mathbf{u}|}$, non-unit vectors are designed in bold, and the

current function is normalized. For the electron source current given by Eq. (19), each comprising a multipole of order (ℓ,m) with a time dependence $e^{i\omega_n t}$, the far-field solutions to Maxwell's equations are given by

$$
\mathbf{B} = -\frac{i}{k}a_M(\ell,m)\nabla\times g_\ell(kr)\mathbf{X}_{\ell,m}
$$

$$
\mathbf{E} = a_M(\ell,m)g_\ell(kr)\mathbf{X}_{\ell,m} \qquad (22)
$$

and the time-averaged power radiated per solid angle $\dfrac{dP(\ell,m)}{d\Omega}$ is

$$
\frac{dP(\ell,m)}{d\Omega} = \frac{c}{8\pi k^2}|a_M(\ell,m)|^2|\mathbf{X}_{\ell,m}|^2 \qquad (23)
$$

where $a_M(\ell,m)$ is

$$
a_M(\ell,m) = \frac{-ek^2}{c\sqrt{\ell(\ell+1)}}\frac{\omega_n}{2\pi}Nj_\ell(kr_n)\Theta\sin(mks) \qquad (24)
$$

In the case that k is the lightlike k^0, then $k = \omega_n/c$, in Eq. (24), and Eqs. (22-23) vanishes for

$$
s = vT_n = R = r_n = \lambda_n \qquad (25)
$$

There is no radiation.

1.D. MAGNETIC FIELD EQUATIONS OF THE ELECTRON

The orbitsphere is a shell of negative charge current comprising correlated charge motion along great circles. For $\ell = 0$, the orbitsphere gives rise to a magnetic moment of 1 Bohr magneton [22]. (The details of the derivation of the magnetic parameters

18

including the electron g factor are given in Ref. [3] and Chp. 1 of Ref. [1].)

$$\mu_B = \frac{e\hbar}{2m_e} = 9.274 \times 10^{-24} \, JT^{-1} \tag{26}$$

The magnetic field of the electron shown in FIGURE 5 is given by

$$\mathbf{H} = \frac{e\hbar}{m_e r_n^3} (\mathbf{i}_r \cos\theta - \mathbf{i}_\theta \sin\theta) \quad \text{for } r < r_n \tag{27}$$

$$\mathbf{H} = \frac{e\hbar}{2m_e r^3} (\mathbf{i}_r 2\cos\theta + \mathbf{i}_\theta \sin\theta) \quad \text{for } r > r_n \tag{28}$$

The energy stored in the magnetic field of the electron is

$$E_{mag} = \frac{1}{2}\mu_o \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} H^2 r^2 \sin\theta \, dr \, d\theta \, d\Phi \tag{29}$$

$$E_{mag\,total} = \frac{\pi\mu_o e^2 \hbar^2}{m_e^2 r_1^3} \tag{30}$$

1.E. STERN-GERLACH EXPERIMENT

The Stern-Gerlach experiment implies a magnetic moment of one Bohr magneton and an associated angular momentum quantum number of 1/2. Historically, this quantum number is called the spin quantum number, s ($s = \frac{1}{2}$; $m_s = \pm\frac{1}{2}$). The superposition of the vector projection of the orbitsphere angular momentum on the z-axis is $\frac{\hbar}{2}$ with an orthogonal component of $\frac{\hbar}{4}$. Excitation of a resonant Larmor precession gives rise to $\hbar$ on an axis $\mathbf{S}$ that precesses about the z-axis called the spin axis at the Larmor frequency at an angle of $\theta = \frac{\pi}{3}$ to give a perpendicular projection of

$$\mathbf{S}_\perp = \hbar\sin\frac{\pi}{3} = \pm\sqrt{\frac{3}{4}}\hbar\,\mathbf{i}_{Y_R} \tag{31}$$

and a projection onto the axis of the applied magnetic field of

$$\mathbf{S}_\| = \hbar\cos\frac{\pi}{3} = \pm\frac{\hbar}{2}\mathbf{i}_z \tag{32}$$

The superposition of the $\frac{\hbar}{2}$, z-axis component of the orbitsphere angular momentum and the $\frac{\hbar}{2}$, z-axis component of $\mathbf{S}$ gives $\hbar$ corresponding to the observed electron magnetic moment of a Bohr magneton, μ_B.

1.F. ELECTRON g FACTOR

Conservation of angular momentum of the orbitsphere permits a discrete change of its "kinetic angular momentum" ($\mathbf{r} \times m\mathbf{v}$) by the applied magnetic field of $\frac{\hbar}{2}$, and concomitantly the "potential angular momentum" ($\mathbf{r} \times e\mathbf{A}$) must change by $-\frac{\hbar}{2}$.

$$\Delta \mathbf{L} = \frac{\hbar}{2} - \mathbf{r} \times e\mathbf{A} \tag{33}$$

$$= \left[\frac{\hbar}{2} - \frac{e\phi}{2\pi} \right] \hat{z} \tag{34}$$

In order that the change of angular momentum, $\Delta \mathbf{L}$, equals zero, ϕ must be $\Phi_0 = \frac{\hbar}{2e}$,

the magnetic flux quantum. The magnetic moment of the electron is parallel or

antiparallel to the applied field only. During the spin-flip transition, power must be

conserved. Power flow is governed by the Poynting power theorem,

$$\nabla \bullet (\mathbf{E} \times \mathbf{H}) = -\frac{\partial}{\partial t} \left[\frac{1}{2} \mu_o \mathbf{H} \bullet \mathbf{H} \right] - \frac{\partial}{\partial t} \left[\frac{1}{2} \varepsilon_o \mathbf{E} \bullet \mathbf{E} \right] - \mathbf{J} \bullet \mathbf{E} \tag{35}$$

Eq. (36) gives the total energy of the flip transition which is the sum of the energy of

reorientation of the magnetic moment (1st term), the magnetic energy (2nd term), the

electric energy (3rd term), and the dissipated energy of a fluxon treading the orbitsphere

(4th term), respectively,

$$\Delta E_{mag}^{spin} = 2\left(1 + \frac{\alpha}{2\pi} + \frac{2}{3}\alpha^2 \left(\frac{\alpha}{2\pi} \right) - \frac{4}{3}\left(\frac{\alpha}{2\pi} \right)^2 \right) \mu_B B \tag{36}$$

$$\Delta E_{mag}^{spin} = g\mu_B B \tag{37}$$

where the stored magnetic energy corresponding to the $\frac{\partial}{\partial t}\left[\frac{1}{2}\mu_o \mathbf{H} \bullet \mathbf{H} \right]$ term increases,

the stored electric energy corresponding to the $\frac{\partial}{\partial t}\left[\frac{1}{2}\varepsilon_o \mathbf{E} \bullet \mathbf{E} \right]$ term increases, and the

$\mathbf{J} \bullet \mathbf{E}$ term is dissipative. The spin-flip transition can be considered as involving a

magnetic moment of g times that of a Bohr magneton. The g factor is redesignated the

fluxon g factor as opposed to the anomalous g factor. Using $\alpha^{-1} = 137.03603(82)$, the

calculated value of $\frac{g}{2}$ is 1.001 159 652 137. The experimental value [23] of $\frac{g}{2}$ is

1.001 159 652 188(4).

<u>1.G. SPIN AND ORBITAL PARAMETERS</u>

The total function that describes the spinning motion of each electron orbitsphere

is composed of two functions. One function, the spin function, is spatially uniform over

the orbitsphere, spins with a quantized angular velocity, and gives rise to spin angular

momentum. The other function, the modulation function, can be spatially uniform—in

which case there is no orbital angular momentum and the magnetic moment of the

electron orbitsphere is one Bohr magneton—or not spatially uniform—in which case

there is orbital angular momentum. The modulation function also rotates with a

quantized angular velocity.

The spin function of the electron corresponds to the nonradiative $n = 1$, $\ell = 0$

20

state of atomic hydrogen which is well known as an s state or orbital. (See FIGURE 1 for the charge function and FIGURE 2 for the current function.) In cases of orbitals of heavier elements and excited states of one electron atoms and atoms or ions of heavier elements with the ℓ quantum number not equal to zero and which are not constant as

5   given by Eq. (14), the constant spin function is modulated by a time and spherical harmonic function as given by Eq. (15) and shown in FIGURE 3. The modulation or traveling charge density wave corresponds to an orbital angular momentum in addition to a spin angular momentum. These states are typically referred to as p, d, f, etc. orbitals. Application of Haus's [16] condition also predicts nonradiation for a constant

10   spin function modulated by a time and spherically harmonic orbital function. There is acceleration without radiation as also shown in Sec. 1.C.c. (Also see Abbott and Griffiths, Goedecke, and Daboul and Jensen [19-21]). However, in the case that such a state arises as an excited state by photon absorption, it is radiative due to a radial dipole term in its current density function since it possesses spacetime Fourier Transform

15   components synchronous with waves traveling at the speed of light [16]. (See Instability of Excited States section of Ref. [1].)

<u>1.G.a Moment of Inertia and Spin and Rotational Energies</u>
        The moments of inertia and the rotational energies as a function of the ℓ

20   quantum number for the solutions of the time-dependent electron charge density functions (Eqs. (14-15)) given in Sec. 1.B are solved using the rigid rotor equation [18]. The details of the derivations of the results as well as the demonstration that Eqs. (14-15) with the results given *infra.* are solutions of the wave equation are given in Chp 1, Rotational Parameters of the Electron (Angular Momentum, Rotational Energy, Moment

25   of Inertia) section of Ref. [1].

$\ell = 0$

$$I_z = I_{spin} = \frac{m_e r_n^2}{2} \tag{38}$$

30    $$L_z = I\omega \mathbf{i}_z = \pm \frac{\hbar}{2} \tag{39}$$

$$E_{rotational} = E_{rotational,\, spin} = \frac{1}{2}\left[I_{spin}\left(\frac{\hbar}{m_e r_n^2}\right)^2 \right] = \frac{1}{2}\left[\frac{m_e r_n^2}{2}\left(\frac{\hbar}{m_e r_n^2}\right)^2 \right] = \frac{1}{4}\left[\frac{\hbar^2}{2 I_{spin}} \right] \tag{40}$$

$$T = \frac{\hbar^2}{2 m_e r_n^2} \tag{41}$$

$\ell \neq 0$

$$I_{orbital} = m_e r_n^2 \left[\frac{\ell(\ell+1)}{\ell^2 + 2\ell + 1} \right]^{\frac{1}{2}} = m_e r_n^2 \sqrt{\frac{\ell}{\ell+1}} \tag{42}$$

$$\mathbf{L} = I\omega \mathbf{i}_z = I_{orbital}\omega \mathbf{i}_z = m_e r_n^2 \left[\frac{\ell(\ell+1)}{\ell^2 + 2\ell + 1} \right]^{\frac{1}{2}} \omega \mathbf{i}_z = m_e r_n^2 \frac{\hbar}{m_e r_n^2} \sqrt{\frac{\ell}{\ell+1}} = \hbar \sqrt{\frac{\ell}{\ell+1}} \tag{43}$$

$$L_{z\,total} = L_{z\,spin} + L_{z\,orbital} \tag{44}$$

$$E_{rotational\ orbital} = \frac{\hbar^2}{2I} \left[\frac{\ell(\ell+1)}{\ell^2 + 2\ell + 1} \right] = \frac{\hbar^2}{2I} \left[\frac{\ell}{\ell+1} \right] = \frac{\hbar^2}{2m_e r_n^2} \left[\frac{\ell}{\ell+1} \right] \tag{45}$$

$$\langle L_{z\,orbital} \rangle = 0 \tag{46}$$

$$\langle E_{rotational\ orbital} \rangle = 0 \tag{47}$$

The orbital rotational energy arises from a spin function (spin angular momentum) modulated by a spherical harmonic angular function (orbital angular momentum). The time-averaged mechanical angular momentum and rotational energy associated with the wave-equation solution comprising a traveling charge-density wave on the orbitsphere is zero as given in Eqs. (46) and (47), respectively. Thus, the principal levels are degenerate except when a magnetic field is applied. In the case of an excited state, the angular momentum of $\hbar$ is carried by the fields of the trapped photon. The amplitudes that couple to external magnetic and electromagnetic fields are given by Eq. (43) and (45), respectively. The rotational energy due to spin is given by Eq. (40), and the total kinetic energy is given by Eq. (41).

1.H. FORCE BALANCE EQUATION

The radius of the nonradiative ($n = 1$) state is solved using the electromagnetic force equations of Maxwell relating the charge and mass density functions wherein the angular momentum of the electron is given by $\hbar$ [1]. The reduced mass arises naturally from an electrodynamic interaction between the electron and the proton of mass m_p.

$$\frac{m_e}{4\pi r_1^2} \frac{v_1^2}{r_1} = \frac{e}{4\pi r_1^2} \frac{Ze}{4\pi\varepsilon_o r_1^2} - \frac{1}{4\pi r_1^2} \frac{\hbar^2}{m_p r_n^3} \tag{48}$$

$$r_1 = \frac{a_H}{Z} \tag{49}$$

where a_H is the radius of the hydrogen atom.

1.I. ENERGY CALCULATIONS

From Maxwell's equations, the potential energy V, kinetic energy T, electric energy or binding energy E_{ele} are

$$V = \frac{-Ze^2}{4\pi\varepsilon_o r_1} = \frac{-Z^2 e^2}{4\pi\varepsilon_o a_H} = -Z^2 X\ 4.3675 \ X\ 10^{-18}\ J = -Z^2 X\ 27.2\ eV \tag{50}$$

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$$T = \frac{Z^2 e^2}{8\pi\varepsilon_o a_H} = Z^2 X 13.59 \ eV \tag{51}$$

$$T = E_{ele} = -\frac{1}{2}\varepsilon_o \int_{\infty}^{r_1} \mathbf{E}^2 dv \quad \text{where } \mathbf{E} = -\frac{Ze}{4\pi\varepsilon_o r^2} \tag{52}$$

$$E_{ele} = -\frac{Ze^2}{8\pi\varepsilon_o r_1} = -\frac{Z^2 e^2}{8\pi\varepsilon_o a_H} = -Z^2 X 2.1786 X 10^{-18} \ J = -Z^2 X 13.598 \ eV \tag{53}$$

The calculated Rydberg constant is $10{,}967{,}758 \ m^{-1}$; the experimental Rydberg constant

is $10{,}967{,}758 \ m^{-1}$. For increasing Z, the velocity becomes a significant fraction of the speed of light; thus, special relativistic corrections were included in the calculation of the ionization energies of one-electron atoms that are given in TABLE I.

TABLE I. Relativistically corrected ionization energies for some one-electron atoms.

One e Atom	Z	γ^* a	Theoretical Ionization Energies (eV) b	Experimental Ionization Energies (eV) c	Relative Difference between Experimental and Calculated d
H	1	1.000007	13.59838	13.59844	0.00000
He^+	2	1.000027	54.40941	54.41778	0.00015
Li^{2+}	3	1.000061	122.43642	122.45429	0.00015
Be^{3+}	4	1.000109	217.68510	217.71865	0.00015
B^{4+}	5	1.000172	340.16367	340.2258	0.00018
C^{5+}	6	1.000251	489.88324	489.99334	0.00022
N^{6+}	7	1.000347	666.85813	667.046	0.00028
O^{7+}	8	1.000461	871.10635	871.4101	0.00035
F^{8+}	9	1.000595	1102.65013	1103.1176	0.00042
Ne^{9+}	10	1.000751	1361.51654	1362.1995	0.00050
Na^{10+}	11	1.000930	1647.73821	1648.702	0.00058
Mg^{11+}	12	1.001135	1961.35405	1962.665	0.00067
Al^{12+}	13	1.001368	2302.41017	2304.141	0.00075
Si^{13+}	14	1.001631	2670.96078	2673.182	0.00083
P^{14+}	15	1.001927	3067.06918	3069.842	0.00090
S^{15+}	16	1.002260	3490.80890	3494.1892	0.00097
Cl^{16+}	17	1.002631	3942.26481	3946.296	0.00102
Ar^{17+}	18	1.003045	4421.53438	4426.2296	0.00106
K^{18+}	19	1.003505	4928.72898	4934.046	0.00108
Ca^{19+}	20	1.004014	5463.97524	5469.864	0.00108
Sc^{20+}	21	1.004577	6027.41657	6033.712	0.00104
Ti^{21+}	22	1.005197	6619.21462	6625.82	0.00100
V^{22+}	23	1.005879	7239.55091	7246.12	0.00091
Cr^{23+}	24	1.006626	7888.62855	7894.81	0.00078
Mn^{24+}	25	1.007444	8566.67392	8571.94	0.00061
Fe^{25+}	26	1.008338	9273.93857	9277.69	0.00040
Co^{26+}	27	1.009311	10010.70111	10012.12	0.00014
Ni^{27+}	28	1.010370	10777.26918	10775.4	-0.00017
Cu^{28+}	29	1.011520	11573.98161	11567.617	-0.00055

a Eq. (1.250) of Ref. [1] (follows Eqs. (6), (16), and (49)).

b Eq. (1.251) of Ref. [1] (Eq. (53) times γ^*).

c From theoretical calculations, interpolation of H isoelectronic and Rydberg series, and experimental data [24-25].

d (Experimental-theoretical)/experimental.

2. TWO ELECTRON ATOMS

Two electron atoms may be solved from a central force balance equation with the nonradiation condition [1]. The centrifugal force, $\mathbf{F}_{centrifugal}$, of each electron is given by

$$\mathbf{F}_{centrifugal} = \frac{m_e \mathbf{v}_n^2}{r_n} \qquad (54)$$

where r_n is the radius of electron n which has velocity $\mathbf{v}_n$. In order to be nonradiative,

24

the velocity for every point on the orbitsphere is given by Eq. (6). Now, consider

electron 1 initially at $r = r_1 = \frac{a_0}{Z}$ (the radius of the one-electron atom of charge Z given

in the Sec. 1.H where $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{e^2 m_e}$ and the spin-nuclear interaction corresponding to the

electron reduced mass is not used here since the electrons have no field at the nucleus

5   upon pairing) and electron 2 initially at $r_n = \infty$. Each electron can be treated as $-e$

charge at the nucleus with $\mathbf{E} = \frac{-e}{4\pi\varepsilon_o r^2}$ for $r > r_n$ and $\mathbf{E} = 0$ for $r < r_n$ where r_n is the

radius of the electron orbitsphere. The centripetal force is the electric force, $\mathbf{F}_{ele}$,

between the electron and the nucleus. Thus, the electric force between electron 2 and the

nucleus is

10    $$\mathbf{F}_{ele(electron\ 2)} = \frac{(Z-1)e^2}{4\pi\varepsilon_o r_2^2} \tag{55}$$

where ε_o is the permittivity of free-space. The second centripetal force, $\mathbf{F}_{mag}$, on the

electron 2 (initially at infinity) from electron 1 (at r_1) is the magnetic force. Due to the

relative motion of the charge-density elements of each electron, a radiation reaction

force arises between the two electrons. This force given in Sections 6.6, 12.10, and 17.3

15   of Jackson [26] achieves the condition that the sum of the mechanical momentum and

electromagnetic momentum is conserved. The magnetic central force is derived from the

Lorentzian force which is relativistically corrected. The magnetic field of electron 2 at

the radius of electron 1 follows from Eq. (1.74b) of Ref. [1] after McQuarrie [22]:

$$\mathbf{B} = \frac{\mu_o e\hbar}{2m_e r_2^3} \tag{56}$$

20   where μ_0 is the permeability of free-space $(4\pi X 10^{-7}\ N / A^2)$. The motion at each point

of electron 1 in the presence of the magnetic field of electron 2 gives rise to a central

force which acts at each point of electron 2. The Lorentzian force density at each point

moving at velocity $\mathbf{v}$ given by Eq. (6) is

$$\mathbf{F}_{mag} = \frac{e}{4\pi r_2^2}\mathbf{v} \times \mathbf{B} \tag{57}$$

25   Substitution of Eq. (6) for $\mathbf{v}$ and Eq. (56) for $\mathbf{B}$ gives

$$\mathbf{F}_{mag} = \frac{1}{4\pi r_2^2}\left[\frac{e^2\mu_o}{2m_e r_1}\right]\frac{\hbar^2}{m_e r_2^3} \tag{58}$$

The term in brackets can be expressed in terms of the fine structure constant α. The

radius of the electron orbitsphere in the $v = c$ frame is λ_C, where $v = c$ corresponds to

the magnetic field front propagation velocity which is the same in all inertial frames, independent of the electron velocity as shown by the velocity addition formula of special relativity [27]. From Eq. (7) and Eqs. (1.144-1.148) of Ref. [1]

$$\frac{e^2 \mu_o}{2 m_e r_1} = 2\pi\alpha\frac{v}{c}$$ (59)

5   where $v = c$. Based on the relativistic invariance of the electron's magnetic moment of a

Bohr magneton $\mu_B = \frac{e\hbar}{2m_e}$ as well as its invariant angular momentum of $\hbar$, it can be

shown that the relativistic correction to Eq. (58) is $\frac{1}{Z}$ times the reciprocal of Eq. (59).

In addition, as given in the Spin Angular Momentum of the Orbitsphere with $\ell = 0$ section of Ref [1], the application of a z-directed magnetic field of electron 2 given by

10   Eq. (1.120) of Ref. [1] to the inner orbitsphere gives rise to a projection of the angular

momentum of electron 1 onto an axis which precesses about the z-axis of $\sqrt{\frac{3}{4}}\hbar$. The

projection of the force between electron 2 and electron 1 is equivalent to that of the angular momentum onto the axis which precesses about the z-axis. Thus, Eq. (58) becomes

15   $$\mathbf{F}_{mag} = \frac{1}{4\pi r_2^2}\frac{1}{Z}\frac{\hbar^2}{m_e r^3}\sqrt{s(s+1)}$$ (60)

Using Eq. (6), the outward centrifugal force on electron 2 is balanced by the electric force and the magnetic force (on electron 2),

$$\frac{m_e}{4\pi r_2^2}\frac{v_2^2}{r_2} = \frac{m_e}{4\pi r_2^2}\frac{\hbar^2}{m_e r_2^3} = \frac{e}{4\pi r_2^2}\frac{(Z-1)e}{4\pi\varepsilon_o r_2^2} + \frac{1}{4\pi r_2^2}\frac{\hbar^2}{Zm_e r_2^3}\sqrt{s(s+1)}$$ (61)

which gives the radius of both electrons as

20   $$r_2 = r_1 = a_0\left(\frac{1}{Z-1} - \frac{\sqrt{s(s+1)}}{Z(Z-1)}\right); s = \frac{1}{2}$$ (62)

(Since the density factor always cancels, it will not be used in subsequent force balance equations).

2.A. IONIZATION ENERGIES CALCULATED USING THE POYNTING POWER
25   ## THEOREM

During ionization, power must be conserved. Power flow is governed by the Poynting power theorem given by Eq. (35). Energy is superposable; thus, the calculation of the ionization energy is determined as a sum of the electric and magnetic contributions. Energy must be supplied to overcome the electric force of the nucleus,

30   and this energy contribution is the negative of the electric work given by Eq. (64).

Additionally, the electrons are initially spin paired at $r_1 = r_2 = 0.566987 a_0$ producing no

magnetic fields; whereas, following ionization, the electrons possess magnetic fields and

corresponding energies. For helium, the contribution to the ionization energy is given as

the energy stored in the magnetic fields of the two electrons at the initial radius where

5 they become spin unpaired. Part of this energy and the corresponding relativistic term

correspond to the precession of the outer electron about the z-axis due to the spin angular

momentum of the inner electron. These terms are the same as those of the corresponding

terms of the hyperfine structure interval of muonium as given in the Muonium Hyperfine

Structure Interval section of Ref [1]. Thus, for helium, which has no electric field

10 beyond r_1 the ionization energy is given by the general formula:

$$Ionization\ Energy(He) = -E(electric) + E(magnetic)\left(1 - \frac{1}{2}\left(\left(\frac{2}{3}\cos\frac{\pi}{3}\right)^2 + \alpha\right)\right)$$

(63)

where,

$$E(electric) = -\frac{(Z-1)e^2}{8\pi\varepsilon_o r_1}$$

(64)

15 $$E(magnetic) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r_1^3} = \frac{8\pi\mu_o \mu_B^2}{r_1^3}$$

(65)

Eq. (65) is derived for each of the two electrons as Eq. (1.129) of the Magnetic

Parameters of the Electron (Bohr Magneton) section of Ref. [1] with the radius given by

Eq. (62).

For $3 \le Z$, a quantized electric field exists for $r > r_1$ that gives rise to a

20 dissipative term, $\mathbf{J} \bullet \mathbf{E}$, of the Poynting Power Vector given by Eq. (35). Thus, the

ionization energies are given by

$$Ionization\ Energy = -Electric\ Energy - \frac{1}{Z}Magnetic\ Energy$$

(66)

With the substitution of the radius given by Eq. (62) into Eq. (6), the velocity v is given

by

25 $$v = \frac{\hbar c}{\sqrt{\left(\frac{4\pi\varepsilon_0 \hbar^2}{e^2} c\left(\frac{1}{Z-1} - \frac{\sqrt{\frac{3}{4}}}{Z(Z-1)}\right)\right)^2 + \hbar^2}} = \frac{\alpha c(Z-1)}{\sqrt{\left(1 - \frac{\sqrt{\frac{3}{4}}}{Z}\right)^2 + \alpha^2(Z-1)^2}}$$

(67)

with $Z > 1$. For increasing Z, the velocity becomes a significant fraction of the speed of

light; thus, special relativistic corrections as given in the Special Relativistic Correction

to the Ionization Energies section of Ref. [1] and Sec. 1.C.a were included in the

calculation of the ionization energies of two-electron atoms given in TABLE II. The calculated ionization energy for helium is 24.58750 eV and the experimental ionization energy is 24.58741 eV. The agreement in the values is within the limit set by experimental error [28].

5   The solution of the helium atom is further proven to be correct since it is used to solve up through twenty-electron atoms in the Three, Four, Five, Six, Seven, Eight, Nine, Ten, Eleven, Twelve, Thirteen, Fifteen, Sixteen, Seventeen, Eighteen, Nineteen, and Twenty-Electron Atoms section of Ref. [1]. The predictions from general solutions for one through twenty-electron atoms are in remarkable agreement with the

10   experimental values known for 400 atoms and ions.

TABLE II. Relativistically corrected ionization energies for some two-electron atoms.

2 e Atom	Z	r_1 (a_o) [a]	Electric Energy [b] (eV)	Magnetic Energy [c] (eV)	Velocity (m/s) [d]	γ^* [e]	Theoretical Ionization Energies [f] (eV)	Experimental Ionization Energies [g] (eV)	Relative Error [h]
He	2	0.566987	23.996467	0.590536	3.85845E+06	1.000021	24.58750	24.58741	-0.000004
Li^+	3	0.35566	76.509	2.543	6.15103E+06	1.00005	75.665	75.64018	-0.0003
Be^{2+}	4	0.26116	156.289	6.423	8.37668E+06	1.00010	154.699	153.89661	-0.0052
B^{3+}	5	0.20670	263.295	12.956	1.05840E+07	1.00016	260.746	259.37521	-0.0053
C^{4+}	6	0.17113	397.519	22.828	1.27836E+07	1.00024	393.809	392.087	-0.0044
N^{5+}	7	0.14605	558.958	36.728	1.49794E+07	1.00033	553.896	552.0718	-0.0033
O^{6+}	8	0.12739	747.610	55.340	1.71729E+07	1.00044	741.023	739.29	-0.0023
F^{7+}	9	0.11297	963.475	79.352	1.93649E+07	1.00057	955.211	953.9112	-0.0014
Ne^{8+}	10	0.10149	1206.551	109.451	2.15560E+07	1.00073	1196.483	1195.8286	-0.0005
Na^{9+}	11	0.09213	1476.840	146.322	2.37465E+07	1.00090	1464.871	1465.121	0.0002
Mg^{10+}	12	0.08435	1774.341	190.652	2.59364E+07	1.00110	1760.411	1761.805	0.0008
Al^{11+}	13	0.07778	2099.05	243.13	2.81260E+07	1.00133	2083.15	2085.98	0.0014
Si^{12+}	14	0.07216	2450.98	304.44	3.03153E+07	1.00159	2433.13	2437.63	0.0018
P^{13+}	15	0.06730	2830.11	375.26	3.25043E+07	1.00188	2810.42	2816.91	0.0023
S^{14+}	16	0.06306	3236.46	456.30	3.46932E+07	1.00221	3215.09	3223.78	0.0027
Cl^{15+}	17	0.05932	3670.02	548.22	3.68819E+07	1.00258	3647.22	3658.521	0.0031
Ar^{16+}	18	0.05599	4130.79	651.72	3.90705E+07	1.00298	4106.91	4120.8857	0.0034
K^{17+}	19	0.05302	4618.77	767.49	4.12590E+07	1.00344	4594.25	4610.8	0.0036
Ca^{18+}	20	0.05035	5133.96	896.20	4.34475E+07	1.00394	5109.38	5128.8	0.0038
Sc^{19+}	21	0.04794	5676.37	1038.56	4.56358E+07	1.00450	5652.43	5674.8	0.0039
Ti^{20+}	22	0.04574	6245.98	1195.24	4.78241E+07	1.00511	6223.55	6249	0.0041
V^{21+}	23	0.04374	6842.81	1366.92	5.00123E+07	1.00578	6822.93	6851.3	0.0041
Cr^{22+}	24	0.04191	7466.85	1554.31	5.22005E+07	1.00652	7450.76	7481.7	0.0041
Mn^{23+}	25	0.04022	8118.10	1758.08	5.43887E+07	1.00733	8107.25	8140.6	0.0041
Fe^{24+}	26	0.03867	8796.56	1978.92	5.65768E+07	1.00821	8792.66	8828	0.0040
Co^{25+}	27	0.03723	9502.23	2217.51	5.87649E+07	1.00917	9507.25	9544.1	0.0039
Ni^{26+}	28	0.03589	10235.12	2474.55	6.09529E+07	1.01022	10251.33	10288.8	0.0036
Cu^{27+}	29	0.03465	10995.21	2750.72	6.31409E+07	1.01136	11025.21	11062.38	0.0034

ᵃ From Eq. (62).

ᵇ From Eq. (64).

ᶜ From Eq. (65).

ᵈ From Eq. (67).

5   ᵉ From Eq. (1.250) of Ref. [1] (follows Eqs. (6), (16), and (49)) with the velocity given by Eq. (67).

ᶠ From Eqs. (63) and (66) with $E(electric)$ of Eq. (64) relativistically corrected by γ^* according to
Eq.(1.251) of Ref. [1] except that the electron-nuclear electrodynamic relativistic factor corresponding to
the reduced mass of Eqs. (1.213-1.223) was not included.

ᵍ From theoretical calculations for ions Ne^{8+} to Cu^{28+} [24-25].

10   ʰ (Experimental-theoretical)/experimental.

The initial central force balance equations with the nonradiation condition, the
initial radii, and the initial energies of the electrons of multi-electron atoms before
excitation is given in R. Mills, *The Grand Unified Theory of Classical Quantum
Mechanics*, January 2005 Edition, BlackLight Power, Inc., Cranbury, New Jersey, (" '05
Mills GUT") and R. L. Mills, "Exact Classical Quantum Mechanical Solutions for One-
Through Twenty-Electron Atoms", submitted; posted at
http://www.blacklightpower.com/pdf/technical/Exact%20Classical%20Quantum%20Me
chanical%20Solutions%20for%20One-%20Through%20Twenty-
Electron%20Atoms%20042204.pdf which are herein incorporated by reference in their
entirety.

3. EXCITED STATES OF HELIUM

(In this section equation numbers of the form (#.#) correspond to those given in
R. Mills, *The Grand Unified Theory of Classical Quantum Mechanics*, January 2005
Edition, BlackLight Power, Inc., Cranbury, New Jersey, (" '05 Mills GUT")).

Bound electrons are described by a charge-density (mass-density) function which
is the product of a radial delta function ($f(r) = \delta(r - r_n)$), two angular functions
(spherical harmonic functions), and a time harmonic function. Thus, a bound electron is
a dynamic "bubble-like" charge-density function. The two-dimensional spherical
surface called an electron orbitsphere can exist in a bound state at only specified
distances from the nucleus. More explicitly, the orbitsphere comprises a two-
dimensional spherical shell of moving charge. The current pattern of the orbitsphere that
gives rise to the phenomenon corresponding to the spin quantum number comprises an
infinite series of correlated orthogonal great circle current loops. As given in the
Orbitsphere Equation of Motion for $\ell = 0$ section, the current pattern (shown in FIGURE
2) is generated over the surface by two orthogonal sets of an infinite series of nested

rotations of two orthogonal great circle current loops where the coordinate axes rotate with the two orthogonal great circles. Each infinitesimal rotation of the infinite series is about the new x-axis and new y-axis which results from the preceding such rotation. For each of the two sets of nested rotations, the angular sum of the rotations about each

5   rotating x-axis and y-axis totals $\sqrt{2}\pi$ radians. The spin function of the electron corresponds to the nonradiative $n = 1$, $\ell = 0$ state which is well known as an s state or orbital. (See FIGURE 1 for the charge function and FIGURE 2 for the current function.) In cases of orbitals of excited states with the ℓ quantum number not equal to zero and which are not constant as given by Eq. (1.64), the constant spin function is modulated by

10   a time and spherical harmonic function as given by Eq. (1.65) and shown in FIGURE 3. The modulation or traveling charge-density wave corresponds to an orbital angular momentum in addition to a spin angular momentum. These states are typically referred to as p, d, f, etc. orbitals.

Each orbitsphere is a spherical shell of negative charge (total charge $= -e$) of

15   zero thickness at a distance r_n from the nucleus (charge $= +Ze$). It is well known that the field of a spherical shell of charge is zero inside the shell and that of a point charge at the origin outside the shell [29] (See FIGURE 1.12 of Ref. [1]). The field of each electron can be treated as that corresponding to a $-e$ charge at the origin with

$$\mathbf{E} = \frac{-e}{4\pi\varepsilon_o r^2} \text{ for } r > r_n \text{ and } \mathbf{E} = 0 \text{ for } r < r_n \text{ where } r_n \text{ is the radius of the electron}$$

20   orbitsphere. Thus, as shown in the Two-Electron Atom section of Ref. [1], the central electric fields due to the helium nucleus are $\mathbf{E} = \frac{2e}{4\pi\varepsilon_o r^2}$ and $\mathbf{E} = \frac{e}{4\pi\varepsilon_o r^2}$ for $r < r_1$ and

$r_1 < r < r_2$, respectively. In the ground state of the helium atom, both electrons are at $r_1 = r_2 = 0.567 a_o$. When a photon is absorbed, one of the initially indistinguishable electrons called electron 1 moves to a smaller radius, and the other called electron 2

25   moves to a greater radius. In the limiting case of the absorption of an ionizing photon, electron 1 moves to the radius of the helium ion, $r_1 = 0.5 a_o$, and electron 2 moves to a continuum radius, $r_2 = \infty$. When a photon is absorbed by the ground state helium atom it generates an effective charge, Z_{P-eff}, within the second orbitsphere such that the electrons move in opposite radial directions while conserving energy and angular

30   momentum. We can determine Z_{P-eff} of the "trapped photon" electric field by requiring that the resonance condition is met for photons of discrete energy, frequency, and

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wavelength for electron excitation in an electromagnetic potential energy well.

It is well known that resonator cavities can trap electromagnetic radiation of discrete resonant frequencies. The orbitsphere is a resonator cavity which traps single photons of discrete frequencies. Thus, photon absorption occurs as an excitation of a resonator mode. The free space photon also comprises a radial Dirac delta function, and the angular momentum of the photon given by $\mathbf{m} = \int \frac{1}{8\pi c} \operatorname{Re}[\mathbf{r} \times (\mathbf{E} \times \mathbf{B}^*)]dx^4 = \hbar$ in the Photon section of Ref. [1] is conserved [30] for the solutions for the resonant photons and excited state electron functions as shown for one-electron atoms in the Excited States of the One-Electron Atom (Quantization) section of Ref. [1]. The correspondence principle holds. That is the change in angular frequency of the electron is equal to the angular frequency of the resonant photon that excites the resonator cavity mode corresponding to the transition, and the energy is given by Planck's equation. It can be demonstrated that the resonance condition between these frequencies is to be satisfied in order to have a net change of the energy field [31].

In general, for a macroscopic multipole with a single m value, a comparison of Eq. (2.33) and Eq. (2.25) shows that the relationship between the angular momentum M_z, energy U, and angular frequency ω is given by Eq. (2.34):

$$\frac{dM_z}{dr} = \frac{m}{\omega} \frac{dU}{dr} \tag{9.1}$$

independent of r where m is an integer. Furthermore, the ratio of the square of the angular momentum, M^2, to the square of the energy, U^2, for a pure (ℓ, m) multipole follows from Eq. (2.25) and Eqs. (2.31-2.33) as given by Eq. (2.35):

$$\frac{M^2}{U^2} = \frac{m^2}{\omega^2} \tag{9.2}$$

From Jackson [32], the quantum mechanical interpretation is that the radiation from such a multipole of order (ℓ, m) carries off $m\hbar$ units of the z component of angular momentum per photon of energy $\hbar\omega$. However, the photon and the electron can each posses only $\hbar$ of angular momentum which requires that Eqs. (9.1-9.2) correspond to a state of the radiation field containing m photons.

As shown in the Excited States of the One-Electron Atom (Quantization) section of Ref. [1] during excitation the spin, orbital, or total angular momentum of the orbitsphere can change by zero or $\pm \hbar$. The selection rules for multipole transitions between quantum states arise from conservation of the photon's multipole moment and

angular momentum of $\hbar$. In an excited state, the time-averaged mechanical angular momentum and rotational energy associated with the traveling charge-density wave on the orbitsphere is zero (Eq. (1.98)), and the angular momentum of $\hbar$ of the photon that excites the electronic state is carried by the fields of the trapped photon. The amplitudes

5   of the rotational energy, moment of inertia, and angular momentum that couple to external magnetic and electromagnetic fields are given by Eq. (1.95) and (1.96), respectively. Furthermore, the electron charge-density waves are nonradiative due to the angular motion as shown in the Appendix I: Nonradiation Based on the Electromagnetic Fields and the Poynting Power Vector section of Ref. [1]. But, excited states are

10   radiative due to a radial dipole that arises from the presence of the trapped photon as shown in the Instability of Excited States section of Ref. [1] corresponding to $m = 1$ in Eqs. (9.1-9.2).

       Then, as shown in the Excited States of the One-Electron Atom (Quantization) section and the Derivation of the Rotational Parameters of the Electron section of Ref.

15   [1], the total number of multipoles, $N_{\ell,s}$, of an energy level corresponding to a principal quantum number n where each multipole corresponds to an ℓ and m $_\ell$ quantum number is

$$N_{\ell,s} = \sum_{\ell=0}^{n-1} \sum_{m_\ell=-\ell}^{+\ell} 1 = \sum_{\ell=0}^{n-1} 2\ell + 1 = (\ell+1)^2 = \ell^2 + 2\ell + 1 = n^2 \qquad (9.3)$$

Any given state may be due to a direct transition or due to the sum of transitions between

20   all intermediate states wherein the multiplicity of possible multipoles increases with higher states. Then, the relationships between the parameters of Eqs. (9.1) and (9.2) due to transitions of quantized angular momentum $\hbar$, energy $\hbar\omega$, and radiative via a radial dipole are given by substitution of $m = 1$ and normalization of the energy U by the total number of degenerate multipoles, n^2. This requires that the photon's electric field

25   superposes that of the nucleus for $r_1 < r < r_2$ such that the radial electric field has a magnitude proportional to e/n at the electron 2 where $n = 2,3,4,...$ for excited states such that U is decreased by the factor of $1/n^2$.

       Energy is conserved between the electric and magnetic energies of the helium atom as shown by Eq. (7.26). The helium atom and the "trapped photon" corresponding

30   to a transition to a resonant excited state have neutral charge and obey Maxwell's equations. Since charge is relativistically invariant, the energies in the electric and magnetic fields of the electrons of the helium atom must be conserved as photons are

emitted or absorbed. The corresponding forces are determined from the requirement that the radial excited-state electric field has a magnitude proportional to e/n at electron 2.

The "trapped photon" is a "standing electromagnetic wave" which actually is a traveling wave that propagates on the surface around the z-axis, and its source current is

5   only at the orbitsphere. The time-function factor, $k(t)$, for the "standing wave" is identical to the time-function factor of the orbitsphere in order to satisfy the boundary (phase) condition at the orbitsphere surface. Thus, the angular frequency of the "trapped photon" has to be identical to the angular frequency of the electron orbitsphere, ω_n, given by Eq. (1.55). Furthermore, the phase condition requires that the angular functions

10   of the "trapped photon" have to be identical to the spherical harmonic angular functions of the electron orbitsphere. Combining $k(t)$ with the ϕ-function factor of the spherical harmonic gives $e^{i(m\phi - \omega_n t)}$ for both the electron and the "trapped photon" function.

The photon "standing wave" in an excited electronic state is a solution of Laplace's equation in spherical coordinates with source currents given by Eq. (2.11)

15   "glued" to the electron and phase-locked to the electron current density wave that travel on the surface with a radial electric field. As given in the Excited States of the One-Electron Atom (Quantization) section of Ref. [1], the photon field is purely radial since the field is traveling azimuthally at the speed of light even though the spherical harmonic function has a velocity less than light speed given by Eq. (1.56). The photon field does

20   not change the nature of the electrostatic field of the nucleus or its energy except at the position of the electron. The photon "standing wave" function comprises a radial Dirac delta function that "samples" the Laplace equation solution only at the position infinitesimally inside of the electron current-density function and superimposes with the proton field to give a field of radial magnitude corresponding to a charge of e/n where

25   $n = 2,3,4,....$.

The electric field of the nucleus for $r_1 < r < r_2$ is

$$\mathbf{E}_{nucleus} = \frac{e}{4\pi\varepsilon_o r^2} \qquad (9.4)$$

From Eq. (2.15), the equation of the electric field of the "trapped photon" for $r = r_2$ where r_2 is the radius of electron 2, is

30   $$\mathbf{E}_{r photon\ n,l,m}\big|_{r=r_2} = \frac{e}{4\pi\varepsilon_o r_2^2}\left[-1 + \frac{1}{n}\left[Y_0^0(\theta,\phi) + \mathrm{Re}\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\}\right]\right]\delta(r - r_n)$$

$$\omega_n = 0 \text{ for } m = 0 \qquad (9.5)$$

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The total central field for $r = r_2$ is given by the sum of the electric field of the nucleus and the electric field of the "trapped photon".

$$\mathbf{E}_{total} = \mathbf{E}_{nucleus} + \mathbf{E}_{photon} \tag{9.6}$$

Substitution of Eqs. (9.4) and (9.5) into Eq. (9.6) gives for $r_1 < r < r_2$,

$$\mathbf{E}_{rtotal} = \frac{e}{4\pi\varepsilon_o r_1^2} + \frac{e}{4\pi\varepsilon_o r_2^2}\left[-1 + \frac{1}{n}\left[Y_0^0(\theta,\phi) + \text{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\}\right]\right]\delta(r - r_n)$$

$$= \frac{1}{n}\frac{e}{4\pi\varepsilon_o r_2^2}\left[Y_0^0(\theta,\phi) + \text{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\}\right]\delta(r - r_n)$$

$$\omega_n = 0 \text{ for } m = 0 \tag{9.7}$$

For $r = r_2$ and $m = 0$, the total radial electric field is

$$\mathbf{E}_{rtotal} = \frac{1}{n}\frac{e}{4\pi\varepsilon_o r^2} \tag{9.8}$$

The result is equivalent to Eq. (2.17) of the Excited States of the One-Electron Atom (Quantization) section of Ref. [1].

In contrast to short comings of quantum-mechanical equations, with classical quantum mechanics (CQM), all excited states of the helium atom can be exactly solved in closed form. The radii of electron 2 are determined from the force balance of the electric, magnetic, and centrifugal forces that corresponds to the minimum of energy of the system. The excited-state energies are then given by the electric energies at these radii. All singlet and triplet states with $\ell = 0$ or $\ell \neq 0$ are solved exactly except for small terms corresponding to the magnetostatic energies in the magnetic fields of excited-state electrons, spin-nuclear interactions, and the very small term due to spin-orbital coupling. In the case of spin-nuclear interactions, a_{He} which includes the reduced electron mass according to Eqs. (1.221-1.224) was used rather than a_0 as a partial correction, and a table of the spin-orbital energies was calculated for $\ell = 1$ to compare to the effect of different ℓ quantum numbers. For over 100 states, the agreement between the predicted and experimental results are remarkable.

3.A SINGLET EXCITED STATES WITH $\ell = 0$ ($1s^2 \rightarrow 1s^1(ns)$)

With $\ell = 0$, the electron source current in the excited state is a constant function given by Eq. (1.64) that spins as a globe about the z-axis:

$$\rho(r,\theta,\phi,t) = \frac{e}{8\pi r^2}[\delta(r - r_n)]\left[Y_0^0(\theta,\phi) + Y_\ell^m(\theta,\phi)\right]$$ (9.9)

As given in the Derivation of the Magnetic Field section in Chapter One of Ref. [1] and by Eq. (12.342), the current is a function of $\sin\theta$ which gives rise to a correction of 2/3 to the field given by Eq. (7.4) and, correspondingly, the magnetic force of two-electron

5   atoms given by Eq. (7.15). The balance between the centrifugal and electric and magnetic forces is given by the Eq. (7.18):

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} + \frac{2}{3}\frac{1}{n}\frac{\hbar^2}{2m_e r_2^3}\sqrt{s(s+1)}$$ (9.10)

with the exceptions that the electric and magnetic forces are reduced by a factor of $\frac{1}{n}$

since the corresponding charge from Eq. (9.8) is $\frac{e}{n}$ and the magnetic force is further

10   corrected by the factor of 2/3. With $s = \frac{1}{2}$,

$$r_2 = \left[n - \frac{\sqrt{\frac{3}{4}}}{3}\right]a_{He} \qquad n = 2,3,4,\ldots$$ (9.11)

The excited-state energy is the energy stored in the electric field, E_{ele}, given by Eqs. (1.232), (1.233), and (10.102) which is the energy of electron 2 relative to the ionized electron at rest having zero energy:

15   $$E_{ele} = -\frac{1}{n}\frac{e^2}{8\pi\varepsilon_o r_2}$$ (9.12)

where r_2 is given by Eq. (9.11) and from Eq. (9.8), $Z = 1/n$ in Eq. (1.233). The energies of the various singlet excited states of helium with $\ell = 0$ appear in TABLE III.

As shown in the Special Relativistic Correction to the Ionization Energies section of Ref. [1] and Sec. 1.C.a the electron possesses an invariant charge-to-mass ratio ($\frac{e}{m_e}$)

20   angular momentum of $\hbar$, and magnetic moment of a Bohr magneton (μ_B). This invariance feature provides for the stability of multielectron atoms as shown in the Two-Electron Atom section of Ref. [1] and the Three, Four, Five, Six, Seven, Eight, Nine, Ten, Eleven, Twelve, Thirteen, Fourteen, Fifteen, Sixteen, Seventeen, Eighteen, Nineteen, and Twenty-Electron Atoms section of Ref. [1]. This feature also permits the

25   existence of excited states wherein electrons magnetically interact. The electron's

motion corresponds to a current which gives rise to a magnetic field with a field strength that is inversely proportional to its radius cubed as given in Eq. (9.10) wherein the magnetic field is a relativistic effect of the electric field as shown by Jackson [33]. Since the forces on electron 2 due to the nucleus and electron 1 (Eq. (9.10)) are radial/central, invariant of r_1, and independent of r_1 with the condition that $r_1 < r_2$, r_2 can be determined without knowledge of r_1. But, once r_2 is determined, r_1 can be solved using the equal and opposite magnetic force of electron 2 on electron 1 and the central Coulombic force corresponding to the nuclear charge of $2e$. Using Eq. (9.10), the force balance between the centrifugal and electric and magnetic forces is

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} - \frac{1}{3n}\frac{\hbar^2}{m_e r_2^3}\sqrt{s(s+1)} \tag{9.13}$$

With $s = \frac{1}{2}$,

$$r_1^3 - \left(\frac{12n}{\sqrt{3}}r_2^3\right)r_1 + \frac{6n}{\sqrt{3}}r_2^3 = 0 \qquad n = 2,3,4,\dots \tag{9.14}$$

where r_2 is given by Eq. (9.11) and r_1 and r_2 are in units of a_{He}. To obtain the solution of cubic Eq. (9.14), let

$$g = \frac{6n}{\sqrt{3}}r_2^3 \tag{9.15}$$

Then, Eq. (9.14) becomes

$$r_1^3 - 2gr_1 + g = 0 \qquad n = 2,3,4,\dots \tag{9.16}$$

and the roots are

$$r_{11} = A + B \tag{9.17}$$

$$r_{12} = -\frac{A+B}{2} + \frac{A-B}{2}\sqrt{-3} \tag{9.18}$$

$$r_{13} = -\frac{A+B}{2} - \frac{A-B}{2}\sqrt{-3} \tag{9.19}$$

where

$$A = \sqrt[3]{-\frac{g}{2} + \sqrt{\frac{g^2}{4} - \frac{8g^3}{27}}} = \sqrt[3]{\frac{g}{2}}\sqrt[3]{z} \tag{9.20}$$

and

$$B = \sqrt[3]{-\frac{g}{2} - \sqrt{\frac{g^2}{4} - \frac{8g^3}{27}}} = \sqrt[3]{\frac{g}{2}}\sqrt[3]{\bar{z}} \tag{9.21}$$

The complex number z is defined by

$$z = -1 + i\sqrt{\frac{32}{27}g - 1} = re^{i\theta} = r(\cos\theta + i\sin\theta) \tag{9.22}$$

where the modulus, r, and argument, θ, are

$$r = \sqrt{\frac{32}{27}g} \tag{9.23}$$

and

$$\theta = \frac{\pi}{2} + \sin^{-1}(1/r) \tag{9.24}$$

respectively. The cube roots are

$$\sqrt[3]{z} = \sqrt[3]{r}e^{i\theta/3} = \sqrt[3]{r}\left(\cos\frac{\theta}{3} + i\sin\frac{\theta}{3}\right) \tag{9.25}$$

$$\sqrt[3]{\bar{z}} = \sqrt[3]{r}e^{-i\theta/3} = \sqrt[3]{r}\left(\cos\frac{\theta}{3} - i\sin\frac{\theta}{3}\right) \tag{9.26}$$

So,

$$A = \sqrt[3]{\frac{g}{2}}r\left(\cos\frac{\theta}{3} + i\sin\frac{\theta}{3}\right) \tag{9.27}$$

and

$$B = \sqrt[3]{\frac{g}{2}}r\left(\cos\frac{\theta}{3} - i\sin\frac{\theta}{3}\right) \tag{9.28}$$

The real and physical root is

$$r_1 = r_{13} = -\sqrt{\frac{2}{3}g}\left(\cos\frac{\theta}{3} - \sqrt{3}\sin\frac{\theta}{3}\right) \tag{9.29}$$

TABLE III. Calculated and experimental energies of He I singlet excited states with $\ell = 0$ $(1s^2 \rightarrow 1s^1(ns))$.

n	r_1 (a_{He}) [a]	r_2 (a_{He}) [b]	Term Symbol	E_{ele} CQM He I Energy Levels [c] (eV)	NIST He I Energy Levels [d] (eV)	Difference CQM-NIST (eV)	Relative Difference [e] (CQM-NIST)
2	0.501820	1.71132	$1s2s\,^1S$	-3.97465	-3.97161	-0.00304	0.00077
3	0.500302	2.71132	$1s3s\,^1S$	-1.67247	-1.66707	-0.00540	0.00324
4	0.500088	3.71132	$1s4s\,^1S$	-0.91637	-0.91381	-0.00256	0.00281
5	0.500035	4.71132	$1s5s\,^1S$	-0.57750	-0.57617	-0.00133	0.00230
6	0.500016	5.71132	$1s6s\,^1S$	-0.39698	-0.39622	-0.00076	0.00193
7	0.500009	6.71132	$1s7s\,^1S$	-0.28957	-0.2891	-0.00047	0.00163
8	0.500005	7.71132	$1s8s\,^1S$	-0.22052	-0.2202	-0.00032	0.00144
9	0.500003	8.71132	$1s9s\,^1S$	-0.17351	-0.1733	-0.00021	0.00124
10	0.500002	9.71132	$1s10s\,^1S$	-0.14008	-0.13992	-0.00016	0.00116
11	0.500001	10.71132	$1s11s\,^1S$	-0.11546	-0.11534	-0.00012	0.00103
					Avg.	**-0.00144**	**0.00175**

[a] Radius of the inner electron 1 from Eq. (9.29).

[b] Radius of the outer electron 2 from Eq. (9.11).

[c] Classical quantum mechanical (CQM) calculated energy levels given by the electric energy (Eq. (9.12)).

[d] Experimental NIST levels [34] with the ionization potential defined as zero.

[e] (Theoretical-Experimental)/Experimental.

3.B TRIPLET EXCITED STATES WITH $\ell = 0$ $(1s^2 \rightarrow 1s^1(ns))$

For $\ell = 0$, time-independent charge-density waves corresponding to the source currents travel on the surface of the orbitsphere of electron 2 about the z-axis at the angular frequency given by Eq. (1.55). In the case of singlet states, the current due to spin of electron 1 and electron 2 rotate in opposite directions; whereas, for triplet states, the relative motion of the spin currents is in the same direction. In the triplet state, the electrons are spin-unpaired, but due to the superposition of the excited state source currents and the current corresponding to the spin-unpairing transition to create the triplet state, the spin-spin force is paramagnetic. The angular momentum corresponding to the excited states is $\hbar$ and the angular momentum change corresponding to the spin-flip or 180° rotation of the Larmor precession is also $\hbar$ as given in the Magnetic Parameters of the Electron (Bohr Magneton) section of Ref. [1]. The maximum projection of the angular momentum of a constant function onto a defined axis (Eq. (1.74a)) is

$$S_{\perp} = \hbar \sin\frac{\pi}{3} = \pm\sqrt{\frac{3}{4}}\hbar\, \mathbf{i}_{Y_R} \tag{9.30}$$

Following the derivation for Eq. (7.15) using Eq. (9.30) and a magnetic moment of $2\mu_B$ corresponding to a total angular momentum of the excited triplet state of $2\hbar$, the spin-spin force for electron 2 is twice that of the singlet states:

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} + 2\frac{2}{3}\frac{1}{n}\frac{\hbar^2}{2m_e r_2^3}\sqrt{s(s+1)} \tag{9.31}$$

With $s = \dfrac{1}{2}$,

$$r_2 = \left[n - \frac{2\sqrt{\frac{3}{4}}}{3}\right]a_{He} \qquad n = 2,3,4,\ldots \tag{9.32}$$

The excited-state energy is the energy stored in the electric field, E_{ele}, given by Eq. (9.12) where r_2 is given by Eq. (9.32). The energies of the various triplet excited states of helium with $\ell = 0$ appear in TABLE IV.

Using r_2 (Eq. (9.32), r_1 can be solved using the equal and opposite magnetic force of electron 2 on electron 1 and the central Coulombic force corresponding to the nuclear charge of $2e$. Using Eq. (9.31), the force balance between the centrifugal and electric and magnetic forces is

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} - \frac{2}{3n}\frac{\hbar^2}{m_e r_2^3}\sqrt{s(s+1)} \tag{9.33}$$

With $s = \dfrac{1}{2}$,

$$r_1^3 - \left(\frac{6n}{\sqrt{3}}r_2^3\right)r_1 + \frac{3n}{\sqrt{3}}r_2^3 = 0 \qquad n = 2,3,4,\ldots \tag{9.34}$$

where r_2 is given by Eq. (9.32) and r_1 and r_2 are in units of a_{He}. To obtain the solution of cubic Eq. (9.34), let

$$g = \frac{3n}{\sqrt{3}}r_2^3 \tag{9.35}$$

Then, Eq. (9.34) becomes

$$r_1^3 - 2gr_1 + g = 0 \qquad n = 2,3,4,\ldots \tag{9.36}$$

Using Eqs. (9.16-9.29), the real and physical root is

$$r_1 = r_{13} = -\sqrt{\frac{2}{3}}g\left(\cos\frac{\theta}{3} - \sqrt{3}\sin\frac{\theta}{3}\right) \tag{9.37}$$

40

TABLE IV. Calculated and experimental energies of He I triplet excited states with $\ell = 0$ ($1s^2 \rightarrow 1s^1(ns)$).

n	r_1 (a_{He}) [a]	r_2 (a_{He}) [b]	Term Symbol	E_{ele} CQM He I Energy Levels [c] (eV)	NIST He I Energy Levels [d] (eV)	Difference CQM-NIST (eV)	Relative Difference [e] (CQM-NIST)
2	0.506514	1.42265	$1s2s\,^3S$	-4.78116	-4.76777	-0.01339	0.00281
3	0.500850	2.42265	$1s3s\,^3S$	-1.87176	-1.86892	-0.00284	0.00152
4	0.500225	3.42265	$1s4s\,^3S$	-0.99366	-0.99342	-0.00024	0.00024
5	0.500083	4.42265	$1s5s\,^3S$	-0.61519	-0.61541	0.00022	-0.00036
6	0.500038	5.42265	$1s6s\,^3S$	-0.41812	-0.41838	0.00026	-0.00063
7	0.500019	6.42265	$1s7s\,^3S$	-0.30259	-0.30282	0.00023	-0.00077
8	0.500011	7.42265	$1s8s\,^3S$	-0.22909	-0.22928	0.00019	-0.00081
9	0.500007	8.42265	$1s9s\,^3S$	-0.17946	-0.17961	0.00015	-0.00083
10	0.500004	9.42265	$1s10s\,^3S$	-0.14437	-0.1445	0.00013	-0.00087
11	0.500003	10.42265	$1s11s\,^3S$	-0.11866	-0.11876	0.00010	-0.00087
					Avg.	**-0.00152**	**-0.00006**

[a] Radius of the inner electron 1 from Eq. (9.37).

[b] Radius of the outer electron 2 from Eq. (9.32).

[e] Classical quantum mechanical (CQM) calculated energy levels given by the electric energy (Eq. (9.12)).

[d] Experimental NIST levels [34] with the ionization potential defined as zero.

[e] (Theoretical-Experimental)/Experimental.

3.C SINGLET EXCITED STATES WITH $\ell \neq 0$

With $\ell \neq 0$, the electron source current in the excited state is the sum of constant and time-dependent functions where the latter, given by Eq. (1.65), travels about the z-axis. The current due to the time dependent term of Eq. (1.65) corresponding to p, d, f, etc. orbitals is

$$\mathbf{J} = \frac{\omega_n}{2\pi}\frac{e}{4\pi r_n^2}N[\delta(r - r_n)]\operatorname{Re}\{Y_\ell^m(\theta,\phi)\}[\mathbf{u}(t) \times \mathbf{r}]$$

$$= \frac{\omega_n}{2\pi}\frac{e}{4\pi r_n^2}N'[\delta(r - r_n)](P_\ell^m(\cos\theta)\cos(m\phi + \omega_n't))[\mathbf{u} \times \mathbf{r}] \qquad (9.38)$$

$$= \frac{\omega_n}{2\pi}\frac{e}{4\pi r_n^2}N'[\delta(r - r_n)](P_\ell^m(\cos\theta)\cos(m\phi + \omega_n't))\sin\theta\hat{\phi}$$

where to keep the form of the spherical harmonic as a traveling wave about the z-axis, $\omega_n' = m\omega_n$ and N and N' are normalization constants. The vectors are defined as

$$\hat{\phi} = \frac{\hat{u} \times \hat{r}}{|\hat{u} \times \hat{r}|} = \frac{\hat{u} \times \hat{r}}{\sin\theta}; \qquad \hat{u} = \hat{z} = orbital\,axis \qquad (9.39)$$

$$\hat{\theta} = \hat{\phi} \times \hat{r} \tag{9.40}$$

"^" denotes the unit vectors $\hat{u} \equiv \dfrac{\mathbf{u}}{|\mathbf{u}|}$, non-unit vectors are designed in bold, and the

current function is normalized.

Jackson [35] gives the general multipole field solution to Maxwell's equations in a source-free region of empty space with the assumption of a time dependence $e^{i\omega_n t}$:

$$\mathbf{B} = \sum_{\ell,m}\left[a_E(\ell,m)f_\ell(kr)\mathbf{X}_{\ell.m} - \frac{i}{k}a_M(\ell,m)\nabla \times g_\ell(kr)\mathbf{X}_{\ell.m}\right]$$

$$\mathbf{E} = \sum_{\ell,m}\left[\frac{i}{k}a_E(\ell,m)\nabla \times f_\ell(kr)\mathbf{X}_{\ell.m} + a_M(\ell,m)g_\ell(kr)\mathbf{X}_{\ell.m}\right] \tag{9.41}$$

where the cgs units used by Jackson are retained in this section. The radial functions $f_\ell(kr)$ and $g_\ell(kr)$ are of the form:

$$\bullet \quad g_\ell(kr) = A_\ell^{(1)}h_\ell^{(1)} + A_\ell^{(2)}h_\ell^{(2)} \tag{9.42}$$

$\mathbf{X}_{\ell.m}$ is the vector spherical harmonic defined by

$$\mathbf{X}_{\ell,m}(\theta,\phi) = \frac{1}{\sqrt{\ell(\ell+1)}}\mathbf{L}Y_{\ell,m}(\theta,\phi) \tag{9.43}$$

where

$$\mathbf{L} = \frac{1}{i}(\mathbf{r} \times \nabla) \tag{9.44}$$

The coefficients $a_E(\ell,m)$ and $a_M(\ell,m)$ of Eq. (9.41) specify the amounts of electric (ℓ,m) multipole and magnetic (ℓ,m) multipole fields, and are determined by sources and boundary conditions as are the relative proportions in Eq. (9.42). Jackson gives the result of the electric and magnetic coefficients from the sources as

$$a_E(\ell,m) = \frac{4\pi k^2}{i\sqrt{\ell(\ell+1)}} \int Y_\ell^{m*}\left\{\rho\frac{\delta}{\delta r}[rj_\ell(kr)] + \frac{ik}{c}(\mathbf{r}\cdot\mathbf{J})j_\ell(kr) - ik\nabla\cdot(r\times M)j_\ell(kr)\right\}d^3x$$

$$\tag{9.45}$$

and

$$a_M(\ell,m) = \frac{-4\pi k^2}{\sqrt{\ell(\ell+1)}} \int j_\ell(kr)Y_\ell^{m*}\mathbf{L}\cdot\left(\frac{\mathbf{J}}{c} + \nabla\times M\right)d^3x \tag{9.46}$$

respectively, where the distribution of charge $\rho(\mathbf{x},t)$, current $\mathbf{J}(\mathbf{x},t)$, and intrinsic magnetization $M(\mathbf{x},t)$ are harmonically varying sources: $\rho(\mathbf{x})e^{-\omega_n t}$, $\mathbf{J}(\mathbf{x})e^{-\omega_n t}$, and $M(\mathbf{x})e^{-\omega_n t}$. From Eq. (9.38), the charge and intrinsic magnetization terms are zero. Since the source dimensions are very small compared to a wavelength ($kr_{max} \ll 1$), the

small argument limit can be used to give the magnetic multipole coefficient $a_M(\ell,m)$ as

$$a_M(\ell,m) = \frac{-4\pi k^{\ell+2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\left(M_{\ell m} + M'_{\ell m}\right) = \frac{-4\pi k^{\ell+2}}{\underbrace{(2\ell+1)!}_{2^n n!}}\left(\frac{\ell+1}{\ell}\right)^{1/2}\left(M_{\ell m} + M'_{\ell m}\right)$$

(9.47)

where the magnetic multipole moments are

$$M_{\ell m} = -\frac{1}{\ell+1}\int r^{\ell} Y^*_{\ell m} \nabla \cdot \left(\frac{\mathbf{r}\times\mathbf{J}}{c}\right)d^3x$$

$$M'_{\ell m} = -\int r^{\ell} Y^*_{\ell m}\nabla\cdot \mathbf{M}d^3x$$

(9.48)

From Eq. (1.108), the geometrical factor of the surface current-density function of the orbitsphere about the z-axis is $\left(\frac{2}{3}\right)^{-1}$. Using the geometrical factor, Eqs. (9.47-9.48), and Eqs. (16.101) and (16.102) of Jackson [36], the multipole coefficient $a_{Mag}(\ell,m)$ of the magnetic force of Eq. (7.15) is

$$a_{Mag}(\ell,m) = \frac{\frac{3}{2}}{(2\ell+1)!!}\frac{1}{\ell+2}\left(\frac{\ell+1}{\ell}\right)^{1/2}$$

(9.49)

For singlet states with $\ell \neq 0$, a minimum energy is achieved with conservation of the photon's angular momentum of $\hbar$ when the magnetic moments of the corresponding angular momenta relative to the electron velocity (and corresponding Lorentzian forces given by Eq. (7.5)) superimpose negatively such that the spin component is radial ($\mathbf{i}_r$-direction) and the orbital component is central ($-\mathbf{i}_r$-direction). The amplitude of the orbital angular momentum $\mathbf{L}_{rotational\ orbital}$, given by Eq. (1.96b) is

$$\mathbf{L} = I\omega\mathbf{i}_z = \hbar\left[\frac{\ell(\ell+1)}{\ell^2+2\ell+1}\right]^{\frac{1}{2}} = \hbar\sqrt{\frac{\ell}{\ell+1}}$$

(9.50)

Thus, using Eqs. (7.15), (9.8), and (9.49-9.50), the magnetic force between the two electrons is

$$\mathbf{F}_{mag} = -\frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\frac{1}{\ell+2}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{2}\frac{\hbar^2}{m_e r^3}\left(\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

(9.51)

and the force balance equation from Eq. (7.18) which achieves the condition that the sum of the mechanical momentum and electromagnetic momentum is conserved as given in Sections 6.6, 12.10, and 17.3 of Jackson [37] is

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n} \frac{e^2}{4\pi\varepsilon_o r_2^2} - \frac{1}{n} \frac{\frac{3}{2}}{(2\ell+1)!!} \left(\frac{\ell+1}{\ell}\right)^{1/2} \frac{1}{\ell+2} \frac{1}{2} \frac{\hbar^2}{m_e r^3} \left(\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

(9.52)

With $s = \dfrac{1}{2}$,

$$r_2 = \left[n + \frac{\frac{3}{4}}{(2\ell+1)!!} \frac{1}{\ell+2} \left(\frac{\ell+1}{\ell}\right)^{1/2} \left(\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right) \right] a_{He} \qquad n = 2,3,4,\ldots \qquad (9.53)$$

5   The excited-state energy is the energy stored in the electric field, E_{ele}, given by Eq.

(9.12) where r_2 is given by Eq. (9.53). The energies of the various singlet excited states

of helium with $\ell \neq 0$ appear in TABLE V.

Using r_2 (Eq. (9.53), r_1 can be solved using the equal and opposite magnetic

force of electron 2 on electron 1 and the central Coulombic force corresponding to the

10   nuclear charge of $2e$. Using Eq. (9.52), the force balance between the centrifugal and

electric and magnetic forces is

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} + \frac{1}{n} \frac{\frac{3}{2}}{(2\ell+1)!!} \left(\frac{\ell+1}{\ell}\right)^{1/2} \frac{1}{\ell+2} \frac{1}{2} \frac{\hbar^2}{m_e r_2^3} \left(\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

(9.54)

With $s = \dfrac{1}{2}$,

$$r_1^3 + \frac{n 8 r_1 r_2^3}{3\left(\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right)} (2\ell+1)!! \left(\frac{\ell}{\ell+1}\right)^{1/2} (\ell+2)$$

$$- \frac{n 4 r_2^3}{3\left(\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right)} (2\ell+1)!! \left(\frac{\ell}{\ell+1}\right)^{1/2} (\ell+2) = 0 \qquad\qquad n = 2,3,4,\ldots$$

(9.55)

where r_2 is given by Eq. (9.53) and r_1 and r_2 are in units of a_{He}. To obtain the solution

of cubic Eq. (9.55), let

$$g = -\frac{n 4 r_2^3}{3\left(\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right)} (2\ell+1)!! \left(\frac{\ell}{\ell+1}\right)^{1/2} (\ell+2) \qquad\qquad (9.56)$$

20   Then, Eq. (9.55) becomes

$$r_1^3 - 2gr_1 + g = 0 \qquad n = 2,3,4,\ldots \tag{9.57}$$

Three distinct cases arise depending on the value of ℓ. For $\ell = 1$ or $\ell = 2$, g of Eq. (9.56) is negative and A and B of Eqs. (9.20) and (9.21), respectively, are real:

$$A = \sqrt[3]{-\frac{g}{2}} \sqrt[3]{1 + \sqrt{1 - \frac{32}{27}g}} \tag{9.58}$$

and

$$B = -\sqrt[3]{-\frac{g}{2}} \sqrt[3]{\sqrt{1 - \frac{32}{27}g} - 1} \tag{9.59}$$

The only real root is

$$r_1 = r_{11} = \sqrt[3]{-\frac{g}{2}} \left\{ \sqrt[3]{1 + \sqrt{1 - \frac{32}{27}g}} - \sqrt[3]{\sqrt{1 - \frac{32}{27}g} - 1} \right\} \tag{9.60}$$

while r_{12} and r_{13} are complex conjugates. When $\ell = 3$ the magnetic force term (2nd term on RHS) of Eq. (9.52) is zero, and the force balance trivially gives

$$r_1 = 0.5a_{He} \tag{9.61}$$

When $\ell = 4,5,6\ldots$ all three roots are real, but, the physical root is r_{13}. In this case, note that $n \geq 5$, $\ell \geq 4$; so, the factor g of Eq. (9.56) is large ($> 10^8$). Expanding r_{13} for large values of g gives

$$r_1 = r_{13} = \frac{1}{2} + \frac{1}{16g} + O\left(g^{-3/2}\right) \tag{9.62}$$

TABLE V. Calculated and experimental energies of He I singlet excited states with $\ell \neq 0$.

n	ℓ	r_1 (a_{He}) [a]	r_2 (a_{He}) [b]	Term Symbol	E_{ele} CQM He I Energy Levels [c] (eV)	NIST He I Energy Levels [d] (eV)	Difference CQM-NIST (eV)	Relative Difference [e] (CQM-NIST)
2	1	0.499929	2.01873	$1s2p\ ^1P^0$	-3.36941	-3.36936	-0.0000477	0.0000141
3	2	0.499999	3.00076	$1s3d\ ^1D$	-1.51116	-1.51331	0.0021542	-0.0014235
3	1	0.499986	3.01873	$1s3p\ ^1P^0$	-1.50216	-1.50036	-0.0017999	0.0011997
4	2	0.500000	4.00076	$1s4d\ ^1D$	-0.85008	-0.85105	0.0009711	-0.0011411
4	3	0.500000	4.00000	$1s4f\ ^1F^0$	-0.85024	-0.85037	0.0001300	-0.0001529
4	1	0.499995	4.01873	$1s4p\ ^1P^0$	-0.84628	-0.84531	-0.0009676	0.0011446
5	2	0.500000	5.00076	$1s5d\ ^1D$	-0.54407	-0.54458	0.0005089	-0.0009345
5	3	0.500000	5.00000	$1s5f\ ^1F^0$	-0.54415	-0.54423	0.0000764	-0.0001404
5	4	0.500000	5.00000	$1s5g\ ^1G$	-0.54415	-0.54417	0.0000159	-0.0000293
5	1	0.499998	5.01873	$1s5p\ ^1P^0$	-0.54212	-0.54158	-0.0005429	0.0010025
6	2	0.500000	6.00076	$1s6d\ ^1D$	-0.37784	-0.37813	0.0002933	-0.0007757
6	3	0.500000	6.00000	$1s6f\ ^1F^0$	-0.37788	-0.37793	0.0000456	-0.0001205
6	4	0.500000	6.00000	$1s6g\ ^1G$	-0.37788	-0.37789	0.0000053	-0.0000140
6	5	0.500000	6.00000	$1s6h\ ^1H^0$	-0.37788	-0.37788	-0.0000045	0.0000119
6	1	0.499999	6.01873	$1s6p\ ^1P^0$	-0.37671	-0.37638	-0.0003286	0.0008730
7	2	0.500000	7.00076	$1s7d\ ^1D$	-0.27760	-0.27779	0.0001907	-0.0006864
7	3	0.500000	7.00000	$1s7f\ ^1F^0$	-0.27763	-0.27766	0.0000306	-0.0001102
7	4	0.500000	7.00000	$1s7g\ ^1G$	-0.27763	-0.27763	0.0000004	-0.0000016
7	5	0.500000	7.00000	$1s7h\ ^1H^0$	-0.27763	-0.27763	0.0000006	-0.0000021
7	6	0.500000	7.00000	$1s7i\ ^1I$	-0.27763	-0.27762	-0.0000094	0.0000338
7	1	0.500000	7.01873	$1s7p\ ^1P^0$	-0.27689	-0.27667	-0.0002186	0.0007900
						Avg.	**0.0000240**	**-0.0000220**

[a] Radius of the inner electron 1 from Eq. (9.60) for $\ell = 1$ or $\ell = 2$, Eq. (9.61) for $\ell = 3$, and Eq. (9.62) for $\ell = 4, 5, 6\ldots$.

[b] Radius of the outer electron 2 from Eq. (9.53).

[e] Classical quantum mechanical (CQM) calculated energy levels given by the electric energy (Eq. (9.12)).

[d] Experimental NIST levels [34] with the ionization potential defined as zero.

[e] (Theoretical-Experimental)/Experimental.

3.D TRIPLET EXCITED STATES WITH $\ell \neq 0$

For triplet states with $\ell \neq 0$, a minimum energy is achieved with conservation of the photon's angular momentum of $\hbar$ when the magnetic moments of the corresponding angular momenta superimpose negatively such that the spin component is central and the orbital component is radial. Furthermore, as given for the triplet states with $\ell = 0$, the spin component in Eqs. (9.51) and (9.52) is doubled. Thus, the force balance equation is

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given by

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} + \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r^3}\left(2\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

(9.63)

With $s = \frac{1}{2}$,

$$r_2 = \left[n - \frac{\frac{3}{4}}{(2\ell+1)!!}\frac{1}{\ell+2}\left(\frac{\ell+1}{\ell}\right)^{1/2}\left(2\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right)\right]a_{He} \qquad n = 2,3,4,\ldots \qquad (9.64)$$

The excited-state energy is the energy stored in the electric field, E_{ele}, given by Eq. (9.12) where r_2 is given by Eq. (9.64). The energies of the various triplet excited states of helium with $\ell \neq 0$ appear in TABLE VI.

Using r_2 (Eq. (9.64), r_1 can be solved using the equal and opposite magnetic force of electron 2 on electron 1 and the central Coulombic force corresponding to the nuclear charge of $2e$. Using Eq. (9.63), the force balance between the centrifugal and electric and magnetic forces is

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} - \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r_2^3}\left(2\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

(9.65)

With $s = \frac{1}{2}$,

$$r_1^3 - \frac{n8r_1 r_2^3}{3\left(\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right)}(2\ell+1)!!\left(\frac{\ell}{\ell+1}\right)^{1/2}(\ell+2)$$

$$+ \frac{n4r_2^3}{3\left(\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right)}(2\ell+1)!!\left(\frac{\ell}{\ell+1}\right)^{1/2}(\ell+2) = 0 \qquad n = 2,3,4,\ldots$$

(9.66)

where r_2 is given by Eq. (9.64) and r_1 and r_2 are in units of a_{He}. To obtain the solution of cubic Eq. (9.66), let

$$g = \frac{n4r_2^3}{3\left(\sqrt{3} - \sqrt{\frac{\ell}{\ell+1}}\right)}(2\ell+1)!\left(\frac{\ell}{\ell+1}\right)^{1/2}(\ell+2) \qquad (9.67)$$

Then, Eq. (9.66) becomes

$$r_1^3 - 2gr_1 + g = 0 \qquad n = 2,3,4,\ldots \qquad (9.68)$$

Using Eqs. (9.16-9.29), the real and physical root is

$$r_1 = r_{13} = -\sqrt{\frac{2}{3}}\, g\left(\cos\frac{\theta}{3} - \sqrt{3}\sin\frac{\theta}{3}\right) \qquad (9.69)$$

TABLE VI. Calculated and experimental energies of He I triplet excited states with $\ell \neq 0$.

n	ℓ	r_1 (a_{He}) [a]	r_2 (a_{He}) [b]	Term Symbol	E_{ele} CQM He I Energy Levels [c] (eV)	NIST He I Energy Levels [d] (eV)	Difference CQM-NIST (eV)	Relative Difference [e] (CQM-NIST)
2	1	0.500571	1.87921	$1s2p\,^3P^0_2$	-3.61957	-3.6233	0.0037349	-0.0010308
2	1	0.500571	1.87921	$1s2p\,^3P^0_1$	-3.61957	-3.62329	0.0037249	-0.0010280
2	1	0.500571	1.87921	$1s2p\,^3P^0_0$	-3.61957	-3.62317	0.0036049	-0.0009949
3	1	0.500105	2.87921	$1s3p\,^3P^0_2$	-1.57495	-1.58031	0.0053590	-0.0033911
3	1	0.500105	2.87921	$1s3p\,^3P^0_1$	-1.57495	-1.58031	0.0053590	-0.0033911
3	1	0.500105	2.87921	$1s3p\,^3P^0_0$	-1.57495	-1.58027	0.0053190	-0.0033659
3	2	0.500011	2.98598	$1s3d\,^3D_3$	-1.51863	-1.51373	-0.0049031	0.0032391
3	2	0.500011	2.98598	$1s3d\,^3D_2$	-1.51863	-1.51373	-0.0049031	0.0032391
3	2	0.500011	2.98598	$1s3d\,^3D_1$	-1.51863	-1.51373	-0.0049031	0.0032391
4	1	0.500032	3.87921	$1s4p\,^3P^0_2$	-0.87671	-0.87949	0.0027752	-0.0031555
4	1	0.500032	3.87921	$1s4p\,^3P^0_1$	-0.87671	-0.87949	0.0027752	-0.0031555
4	1	0.500032	3.87921	$1s4p\,^3P^0_0$	-0.87671	-0.87948	0.0027652	-0.0031442
4	2	0.500003	3.98598	$1s4d\,^3D_3$	-0.85323	-0.85129	-0.0019398	0.0022787
4	2	0.500003	3.98598	$1s4d\,^3D_2$	-0.85323	-0.85129	-0.0019398	0.0022787
4	2	0.500003	3.98598	$1s4d\,^3D_1$	-0.85323	-0.85129	-0.0019398	0.0022787
4	3	0.500000	3.99857	$1s4f\,^3F^0_3$	-0.85054	-0.85038	-0.0001638	0.0001926
4	3	0.500000	3.99857	$1s4f\,^3F^0_4$	-0.85054	-0.85038	-0.0001638	0.0001926
4	3	0.500000	3.99857	$1s4f\,^3F^0_2$	-0.85054	-0.85038	-0.0001638	0.0001926
5	1	0.500013	4.87921	$1s5p\,^3P^0_2$	-0.55762	-0.55916	0.0015352	-0.0027456
5	1	0.500013	4.87921	$1s5p\,^3P^0_1$	-0.55762	-0.55916	0.0015352	-0.0027456
5	1	0.500013	4.87921	$1s5p\,^3P^0_0$	-0.55762	-0.55915	0.0015252	-0.0027277
5	2	0.500001	4.98598	$1s5d\,^3D_3$	-0.54568	-0.54472	-0.0009633	0.0017685
5	2	0.500001	4.98598	$1s5d\,^3D_2$	-0.54568	-0.54472	-0.0009633	0.0017685
5	2	0.500001	4.98598	$1s5d\,^3D_1$	-0.54568	-0.54472	-0.0009633	0.0017685
5	3	0.500000	4.99857	$1s5f\,^3F^0_3$	-0.54431	-0.54423	-0.0000791	0.0001454
5	3	0.500000	4.99857	$1s5f\,^3F^0_4$	-0.54431	-0.54423	-0.0000791	0.0001454
5	3	0.500000	4.99857	$1s5f\,^3F^0_2$	-0.54431	-0.54423	-0.0000791	0.0001454
5	4	0.500000	4.99988	$1s5g\,^3G_4$	-0.54417	-0.54417	0.0000029	-0.0000054

5	4	0.500000	4.99988	1s5g ^{3}G$_5$	-0.54417	-0.54417	0.0000029	-0.0000054
5	4	0.500000	4.99988	1s5g ^{3}G$_3$	-0.54417	-0.54417	0.0000029	-0.0000054
6	1	0.500006	5.87921	1s6p ^{3}P$^0_2$	-0.38565	-0.38657	0.0009218	-0.0023845
6	1	0.500006	5.87921	1s6p ^{3}P$^0_1$	-0.38565	-0.38657	0.0009218	-0.0023845
6	1	0.500006	5.87921	1s6p ^{3}P$^0_0$	-0.38565	-0.38657	0.0009218	-0.0023845
6	2	0.500001	5.98598	1s6d ^{3}D$_3$	-0.37877	-0.37822	-0.0005493	0.0014523
6	2	0.500001	5.98598	1s6d ^{3}D$_2$	-0.37877	-0.37822	-0.0005493	0.0014523
6	2	0.500001	5.98598	1s6d ^{3}D$_1$	-0.37877	-0.37822	-0.0005493	0.0014523
6	3	0.500000	5.99857	1s6f ^{3}F$^0_3$	-0.37797	-0.37793	-0.0000444	0.0001176
6	3	0.500000	5.99857	1s6f ^{3}F$^0_4$	-0.37797	-0.37793	-0.0000444	0.0001176
6	3	0.500000	5.99857	1s6f ^{3}F$^0_2$	-0.37797	-0.37793	-0.0000444	0.0001176
6	4	0.500000	5.99988	1s6g ^{3}G$_4$	-0.37789	-0.37789	-0.0000023	0.0000060
6	4	0.500000	5.99988	1s6g ^{3}G$_5$	-0.37789	-0.37789	-0.0000023	0.0000060
6	4	0.500000	5.99988	1s6g ^{3}G$_3$	-0.37789	-0.37789	-0.0000023	0.0000060
6	5	0.500000	5.99999	1s6h ^{3}H$^0_4$	-0.37789	-0.37788	-0.0000050	0.0000133
6	5	0.500000	5.99999	1s6h ^{3}H$^0_5$	-0.37789	-0.37788	-0.0000050	0.0000133
6	5	0.500000	5.99999	1s6h ^{3}H$^0_6$	-0.37789	-0.37788	-0.0000050	0.0000133
7	1	0.500003	6.87921	1s7p ^{3}P$^0_2$	-0.28250	-0.28309	0.0005858	-0.0020692
7	1	0.500003	6.87921	1s7p ^{3}P$^0_1$	-0.28250	-0.28309	0.0005858	-0.0020692
7	1	0.500003	6.87921	1s7p ^{3}P$^0_0$	-0.28250	-0.28309	0.0005858	-0.0020692
7	2	0.500000	6.98598	1s7d ^{3}D$_3$	-0.27819	-0.27784	-0.0003464	0.0012468
7	2	0.500000	6.98598	1s7d ^{3}D$_2$	-0.27819	-0.27784	-0.0003464	0.0012468
7	2	0.500000	6.98598	1s7d ^{3}D$_1$	-0.27819	-0.27784	-0.0003464	0.0012468
7	3	0.500000	6.99857	1s7f ^{3}F$^0_3$	-0.27769	-0.27766	-0.0000261	0.0000939
7	3	0.500000	6.99857	1s7f ^{3}F$^0_4$	-0.27769	-0.27766	-0.0000261	0.0000939
7	3	0.500000	6.99857	1s7f ^{3}F$^0_2$	-0.27769	-0.27766	-0.0000261	0.0000939
7	4	0.500000	6.99988	1s7g ^{3}G$_4$	-0.27763	-0.27763	-0.0000043	0.0000155
7	4	0.500000	6.99988	1s7g ^{3}G$_5$	-0.27763	-0.27763	-0.0000043	0.0000155
7	4	0.500000	6.99988	1s7g ^{3}G$_3$	-0.27763	-0.27763	-0.0000043	0.0000155
7	5	0.500000	6.99999	1s7h ^{3}H$^0_5$	-0.27763	-0.27763	0.0000002	-0.0000009

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7	5	0.500000	6.99999	1s7h $^3H^0_6$	-0.27763	-0.27763	0.0000002	-0.0000009
7	5	0.500000	6.99999	1s7h $^3H^0_4$	-0.27763	-0.27763	0.0000002	-0.0000009
7	6	0.500000	7.00000	1s7 i^3I_5	-0.27763	-0.27762	-0.0000094	0.0000339
7	6	0.500000	7.00000	1s7i $^3I6$	-0.27763	-0.27762	-0.0000094	0.0000339
7	6	0.500000	7.00000	1s7i $^3I_7$	-0.27763	-0.27762	-0.0000094	0.0000339
						Avg.	**0.0002768**	**-0.0001975**

a Radius of the inner electron 1 from Eq. (9.69).

b Radius of the outer electron 2 from Eq. (9.64).

c Classical quantum mechanical (CQM) calculated energy levels given by the electric energy (Eq. (9.12)).

d Experimental NIST levels [34] with the ionization potential defined as zero.

e (Theoretical-Experimental)/Experimental.

3.E ALL EXCITED HE I STATES

The combined energies of the various states of helium appear in TABLE VII. A plot of the predicted and experimental energies of levels assigned by NIST [34] appears in FIGURE 6. For over 100 states, the r-squared value is 0.999994, and the typical average relative difference is about 5 significant figures which is within the error of the experimental data. The agreement is remarkable.

The hydrino states given in the Hydrino Theory—BlackLight Process section of Ref. [1] are strongly supported by the calculation of the helium excited states as well as the hydrogen excited states given in the Excited States of the One-Electron Atom (Quantization) section of Ref. [1] since the electron-photon model is the same in both the excited states and in the lower-energy states of hydrogen except that the photon provides a central field of magnitude n in the hydrino case and $1/n$ in the excited-state case.

TABLE VII. Calculated and experimental energies of states of helium.

n	ℓ	r_1 (a_{He}) a	r_2 (a_{He}) b	Term Symbol	E_{ele} CQM He I Energy Levels c (eV)	NIST He I Energy Levels d (eV)	Difference CQM-NIST (eV)	Relative Difference e (CQM-NIST)
1	0	0.56699	0.566987	1s2 ^1S	-24.58750	-24.58741	-0.000092	0.0000038
2	0	0.506514	1.42265	1s2s ^3S	-4.78116	-4.76777	-0.0133929	0.0028090
2	0	0.501820	1.71132	1s2s ^1S	-3.97465	-3.97161	-0.0030416	0.0007658
2	1	0.500571	1.87921	1s2p $^3P^0_2$	-3.61957	-3.6233	0.0037349	-0.0010308
2	1	0.500571	1.87921	1s2p $^3P^0_1$	-3.61957	-3.62329	0.0037249	-0.0010280
2	1	0.500571	1.87921	1s2p $^3P^0_0$	-3.61957	-3.62317	0.0036049	-0.0009949
2	1	0.499929	2.01873	1s2p $^1P^0$	-3.36941	-3.36936	-0.0000477	0.0000141
3	0	0.500850	2.42265	1s3s ^3S	-1.87176	-1.86892	-0.0028377	0.0015184
3	0	0.500302	2.71132	1s3s ^1S	-1.67247	-1.66707	-0.0054014	0.0032401

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3	1	0.500105	2.87921	$1s3p\ ^3P^0_2$	-1.57495	-1.58031	0.0053590	-0.0033911
3	1	0.500105	2.87921	$1s3p\ ^3P^0_1$	-1.57495	-1.58031	0.0053590	-0.0033911
3	1	0.500105	2.87921	$1s3p\ ^3P^0_0$	-1.57495	-1.58027	0.0053190	-0.0033659
3	2	0.500011	2.98598	$1s3d\ ^3D_3$	-1.51863	-1.51373	-0.0049031	0.0032391
3	2	0.500011	2.98598	$1s3d\ ^3D_2$	-1.51863	-1.51373	-0.0049031	0.0032391
3	2	0.500011	2.98598	$1s3d\ ^3D_1$	-1.51863	-1.51373	-0.0049031	0.0032391
3	2	0.499999	3.00076	$1s3d\ ^1D$	-1.51116	-1.51331	0.0021542	-0.0014235
3	1	0.499986	3.01873	$1s3p\ ^1P^0$	-1.50216	-1.50036	-0.0017999	0.0011997
4	0	0.500225	3.42265	$1s4s\ ^3S$	-0.99366	-0.99342	-0.0002429	0.0002445
4	0	0.500088	3.71132	$1s4s\ ^1S$	-0.91637	-0.91381	-0.0025636	0.0028054
4	1	0.500032	3.87921	$1s4p\ ^3P^0_2$	-0.87671	-0.87949	0.0027752	-0.0031555
4	1	0.500032	3.87921	$1s4p\ ^3P^0_1$	-0.87671	-0.87949	0.0027752	-0.0031555
4	1	0.500032	3.87921	$1s4p\ ^3P^0_0$	-0.87671	-0.87948	0.0027652	-0.0031442
4	2	0.500003	3.98598	$1s4d\ ^3D_3$	-0.85323	-0.85129	-0.0019398	0.0022787
4	2	0.500003	3.98598	$1s4d\ ^3D_2$	-0.85323	-0.85129	-0.0019398	0.0022787
4	2	0.500003	3.98598	$1s4d\ ^3D_1$	-0.85323	-0.85129	-0.0019398	0.0022787
4	2	0.500000	4.00076	$1s4d\ ^1D$	-0.85008	-0.85105	0.0009711	-0.0011411
4	3	0.500000	3.99857	$1s4f\ ^3F^0_3$	-0.85054	-0.85038	-0.0001638	0.0001926
4	3	0.500000	3.99857	$1s4f\ ^3F^0_4$	-0.85054	-0.85038	-0.0001638	0.0001926
4	3	0.500000	3.99857	$1s4f\ ^3F^0_2$	-0.85054	-0.85038	-0.0001638	0.0001926
4	3	0.500000	4.00000	$1s4f\ ^1F^0$	-0.85024	-0.85037	0.0001300	-0.0001529
4	1	0.499995	4.01873	$1s4p\ ^1P^0$	-0.84628	-0.84531	-0.0009676	0.0011446
5	0	0.500083	4.42265	$1s5s\ ^3S$	-0.61519	-0.61541	0.0002204	-0.0003582
5	0	0.500035	4.71132	$1s5s\ ^1S$	-0.57750	-0.57617	-0.0013253	0.0023002
5	1	0.500013	4.87921	$1s5p\ ^3P^0_2$	-0.55762	-0.55916	0.0015352	-0.0027456
5	1	0.500013	4.87921	$1s5p\ ^3P^0_1$	-0.55762	-0.55916	0.0015352	-0.0027456
5	1	0.500013	4.87921	$1s5p\ ^3P^0_0$	-0.55762	-0.55915	0.0015252	-0.0027277
5	2	0.500001	4.98598	$1s5d\ ^3D_3$	-0.54568	-0.54472	-0.0009633	0.0017685
5	2	0.500001	4.98598	$1s5d\ ^3D_2$	-0.54568	-0.54472	-0.0009633	0.0017685
5	2	0.500001	4.98598	$1s5d\ ^3D_1$	-0.54568	-0.54472	-0.0009633	0.0017685
5	2	0.500000	5.00076	$1s5d\ ^1D$	-0.54407	-0.54458	0.0005089	-0.0009345
5	3	0.500000	4.99857	$1s5f\ ^3F^0_3$	-0.54431	-0.54423	-0.0000791	0.0001454
5	3	0.500000	4.99857	$1s5f\ ^3F^0_4$	-0.54431	-0.54423	-0.0000791	0.0001454
5	3	0.500000	4.99857	$1s5f\ ^3F^0_2$	-0.54431	-0.54423	-0.0000791	0.0001454
5	3	0.500000	5.00000	$1s5f\ ^1F^0$	-0.54415	-0.54423	0.0000764	-0.0001404
5	4	0.500000	4.99988	$1s5g\ ^3G_4$	-0.54417	-0.54417	0.0000029	-0.0000054
5	4	0.500000	4.99988	$1s5g\ ^3G_5$	-0.54417	-0.54417	0.0000029	-0.0000054
5	4	0.500000	4.99988	$1s5g\ ^3G_3$	-0.54417	-0.54417	0.0000029	-0.0000054
5	4	0.500000	5.00000	$1s5g\ ^1G$	-0.54415	-0.54417	0.0000159	-0.0000293
5	1	0.499998	5.01873	$1s5p\ ^1P^0$	-0.54212	-0.54158	-0.0005429	0.0010025
6	0	0.500038	5.42265	$1s6s\ ^3S$	-0.41812	-0.41838	0.0002621	-0.0006266

6	0	0.500016	5.71132	$1s6s\,^1S$	-0.39698	-0.39622	-0.0007644	0.0019291
6	1	0.500006	5.87921	$1s6p\,^3P^0_2$	-0.38565	-0.38657	0.0009218	-0.0023845
6	1	0.500006	5.87921	$1s6p\,^3P^0_1$	-0.38565	-0.38657	0.0009218	-0.0023845
6	1	0.500006	5.87921	$1s6p\,^3P^0_0$	-0.38565	-0.38657	0.0009218	-0.0023845
6	2	0.500001	5.98598	$1s6d\,^3D_3$	-0.37877	-0.37822	-0.0005493	0.0014523
6	2	0.500001	5.98598	$1s6d\,^3D_2$	-0.37877	-0.37822	-0.0005493	0.0014523
6	2	0.500001	5.98598	$1s6d\,^3D_1$	-0.37877	-0.37822	-0.0005493	0.0014523
6	2	0.500000	6.00076	$1s6d\,^1D$	-0.37784	-0.37813	0.0002933	-0.0007757
6	3	0.500000	5.99857	$1s6f\,^3F^0_3$	-0.37797	-0.37793	-0.0000444	0.0001176
6	3	0.500000	5.99857	$1s6f\,^3F^0_4$	-0.37797	-0.37793	-0.0000444	0.0001176
6	3	0.500000	5.99857	$1s6f\,^3F^0_2$	-0.37797	-0.37793	-0.0000444	0.0001176
6	3	0.500000	6.00000	$1s6f\,^1F^0$	-0.37788	-0.37793	0.0000456	-0.0001205
6	4	0.500000	5.99988	$1s6g\,^3G_4$	-0.37789	-0.37789	-0.0000023	0.0000060
6	4	0.500000	5.99988	$1s6g\,^3G_5$	-0.37789	-0.37789	-0.0000023	0.0000060
6	4	0.500000	5.99988	$1s6g\,^3G_3$	-0.37789	-0.37789	-0.0000023	0.0000060
6	4	0.500000	6.00000	$1s6g\,^1G$	-0.37788	-0.37789	0.0000053	-0.0000140
6	5	0.500000	5.99999	$1s6h\,^3H^0_4$	-0.37789	-0.37788	-0.0000050	0.0000133
6	5	0.500000	5.99999	$1s6h\,^3H^0_5$	-0.37789	-0.37788	-0.0000050	0.0000133
6	5	0.500000	5.99999	$1s6h\,^3H^0_6$	-0.37789	-0.37788	-0.0000050	0.0000133
6	5	0.500000	6.00000	$1s6h\,^1H^0$	-0.37788	-0.37788	-0.0000045	0.0000119
6	1	0.499999	6.01873	$1s6p\,^1P^0$	-0.37671	-0.37638	-0.0003286	0.0008730
7	0	0.500019	6.42265	$1s7s\,^3S$	-0.30259	-0.30282	0.0002337	-0.0007718
7	0	0.500009	6.71132	$1s7s\,^1S$	-0.28957	-0.2891	-0.0004711	0.0016295
7	1	0.500003	6.87921	$1s7p\,^3P^0_2$	-0.28250	-0.28309	0.0005858	-0.0020692
7	1	0.500003	6.87921	$1s7p\,^3P^0_1$	-0.28250	-0.28309	0.0005858	-0.0020692
7	1	0.500003	6.87921	$1s7p\,^3P^0_0$	-0.28250	-0.28309	0.0005858	-0.0020692
7	2	0.500000	6.98598	$1s7d\,^3D_3$	-0.27819	-0.27784	-0.0003464	0.0012468
7	2	0.500000	6.98598	$1s7d\,^3D_2$	-0.27819	-0.27784	-0.0003464	0.0012468
7	2	0.500000	6.98598	$1s7d\,^3D_1$	-0.27819	-0.27784	-0.0003464	0.0012468
7	2	0.500000	7.00076	$1s7d\,^1D$	-0.27760	-0.27779	0.0001907	-0.0006864
7	3	0.500000	6.99857	$1s7f\,^3F^0_3$	-0.27769	-0.27766	-0.0000261	0.0000939
7	3	0.500000	6.99857	$1s7f\,^3F^0_4$	-0.27769	-0.27766	-0.0000261	0.0000939
7	3	0.500000	6.99857	$1s7f\,^3F^0_2$	-0.27769	-0.27766	-0.0000261	0.0000939
7	3	0.500000	7.00000	$1s7f\,^1F^0$	-0.27763	-0.27766	0.0000306	-0.0001102
7	4	0.500000	6.99988	$1s7g\,^3G_4$	-0.27763	-0.27763	-0.0000043	0.0000155
7	4	0.500000	6.99988	$1s7g\,^3G_5$	-0.27763	-0.27763	-0.0000043	0.0000155
7	4	0.500000	6.99988	$1s7g\,^3G_3$	-0.27763	-0.27763	-0.0000043	0.0000155
7	4	0.500000	7.00000	$1s7g\,^1G$	-0.27763	-0.27763	0.0000004	-0.0000016
7	5	0.500000	6.99999	$1s7h\,^3H^0_5$	-0.27763	-0.27763	0.0000002	-0.0000009
7	5	0.500000	6.99999	$1s7h\,^3H^0_6$	-0.27763	-0.27763	0.0000002	-0.0000009

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7	5	0.500000	6.99999	$1s7h\ {}^3H^{\circ}_4$	-0.27763	-0.27763	0.0000002	-0.0000009
7	5	0.500000	7.00000	$1s7h\ {}^1H^0$	-0.27763	-0.27763	0.0000006	-0.0000021
7	6	0.500000	7.00000	$1s7\ i^3I_5$	-0.27763	-0.27762	-0.0000094	0.0000339
7	6	0.500000	7.00000	$1s7i\ {}^3I6$	-0.27763	-0.27762	-0.0000094	0.0000339
7	6	0.500000	6.78349	$1s7i\ {}^3I_7$	-0.27763	-0.27762	-0.0000094	0.0000339
7	6	0.500000	7.00000	$1s7i\ {}^1I$	-0.27763	-0.27762	-0.0000094	0.0000338
7	1	0.500000	7.01873	$1s7p\ {}^1P^0$	-0.27689	-0.27667	-0.0002186	0.0007900
8	0	0.500011	7.42265	$1s8s\ {}^3S$	-0.22909	-0.22928	0.0001866	-0.0008139
8	0	0.500005	7.71132	$1s8s\ {}^1S$	-0.22052	-0.2202	-0.0003172	0.0014407
9	0	0.500007	8.42265	$1s9s\ {}^3S$	-0.17946	-0.17961	0.0001489	-0.0008291
9	0	0.500003	8.71132	$1s9s\ {}^1S$	-0.17351	-0.1733	-0.0002141	0.0012355
10	0	0.500004	9.42265	$1s10s\ {}^3S$	-0.14437	-0.1445	0.0001262	-0.0008732
10	0	0.500002	9.71132	$1s10s\ {}^1S$	-0.14008	-0.13992	-0.0001622	0.0011594
11	0	0.500003	10.42265	$1s11s\ {}^3S$	-0.11866	-0.11876	0.0001037	-0.0008734
11	0	0.500001	10.71132	$1s11s\ {}^1S$	-0.11546	-0.11534	-0.0001184	0.0010268
						Avg.	**-0.000112**	**0.0000386**

[a] Radius of the inner electron 1 of singlet excited states with $\ell = 0$ from Eq. (9.29); triplet excited states with $\ell = 0$ from Eq. (9.37); singlet excited states with $\ell \neq 0$ from Eq. (9.60) for $\ell = 1$ or $\ell = 2$ and Eq. (9.61) for $\ell = 3$, and Eq. (9.62) for $\ell = 4,5,6...$; triplet excited states with $\ell \neq 0$ from Eq. (9.69), and $1s^2\ {}^1S$ from Eq. (7.19).

[b] Radius of the outer electron 2 of singlet excited states with $\ell = 0$ from Eq. (9.11); triplet excited states with $\ell = 0$ from Eq. (9.32); singlet excited states with $\ell \neq 0$ from Eq. (9.53); triplet excited states with $\ell \neq 0$ from Eq. (9.64), and $1s^2\ {}^1S$ from Eq. (7.19).

[c] Classical quantum mechanical (CQM) calculated excited-state energy levels given by the electric energy (Eq. (9.12)) and the energy level of $1s^2\ {}^1S$ is given by Eqs. (7.28-7.30).

[d] Experimental NIST levels [34] with the ionization potential defined as zero.

[e] (Theoretical-Experimental)/Experimental.

3.F SPIN-ORBITAL COUPLING OF EXCITED STATES WITH $\ell \neq 0$

Due to 1.) the invariance of each of $\dfrac{e}{m_e}$ of the electron, the electron angular

momentum of $\hbar$, and μ_B from the spin angular and orbital angular momentum, 2.) the condition that flux must be linked by the electron orbitsphere in units of the magnetic flux quantum, and 3.) the maximum projection of the spin angular momentum of the electron onto an axis is $\sqrt{\dfrac{3}{4}}\hbar$, the magnetic energy term of the electron g-factor gives the spin-orbital coupling energy $E_{s/o}$ (Eq. (2.102)):

$$E_{s/o} = 2\frac{\alpha}{2\pi}\left(\frac{e\hbar}{2m_e}\right)\frac{\mu_0 e\hbar}{2(2\pi m_e)\left(\frac{r}{2\pi}\right)^3}\sqrt{\frac{3}{4}} = \frac{\alpha\pi\mu_0 e^2\hbar^2}{m_e^2 r^3}\sqrt{\frac{3}{4}} \qquad (9.70)$$

For the $n = 2$ state of hydrogen, the radius is $r = 2a_0$, and the predicted energy

difference between the $^2P_{3/2}$ and $^2P_{1/2}$ levels of the hydrogen atom due to spin-orbital

interaction is

$$E_{s/o} = \frac{\alpha \pi \mu_0 e^2 \hbar^2}{8 m_e^2 a_0^3} \sqrt{\frac{3}{4}} \qquad\qquad (9.71)$$

As in the case of the $^2P_{1/2} \rightarrow {}^2S_{1/2}$ transition, the photon-momentum transfer for the

5    $^2P_{3/2} \rightarrow {}^2P_{1/2}$ transition gives rise to a small frequency shift derived after that of the Lamb

shift with $\Delta m_\ell = -1$ included. The energy, E_{FS}, for the $^2P_{3/2} \rightarrow {}^2P_{1/2}$ transition called the

fine structure splitting is given by Eq. (2.113):

$$E_{FS} = \frac{\alpha^5 (2\pi)^2}{8} m_e c^2 \sqrt{\frac{3}{4}} + \left(13.5983 \ eV \left(1 - \frac{1}{2^2} \right) \right)^2 \left[\frac{\left(\frac{3}{4\pi} \left(1 - \sqrt{\frac{3}{4}} \right) \right)^2}{2 h \mu_e c^2} + \frac{\left(1 + \left(1 - \sqrt{\frac{3}{4}} \right) \right)^2}{2 h m_H c^2} \right]$$

$$= 4.5190 \ X \ 10^{-5} \ eV + 1.75407 \ X \ 10^{-7} \ eV$$

$$= 4.53659 \ X \ 10^{-5} \ eV$$

(9.72)

10   where the first term corresponds to $E_{s/o}$ given by Eq. (9.71) expressed in terms of the

mass energy of the electron (Eq. (2.106)) and the second and third terms correspond to

the electron recoil and atom recoil, respectively. The energy of $4.53659 \ X \ 10^{-5} \ eV$

corresponds to a frequency of $10,969.4 \ MHz$ or a wavelength of $2.73298 \ cm$. The

experimental value of the $^2P_{3/2} - {}^2P_{1/2}$ transition frequency is $10,969.1 \ MHz$. The large

15   natural widths of the hydrogen $2p$ levels limits the experimental accuracy; yet, given

this limitation, the agreement between the theoretical and experimental fine structure is

excellent. Using r_2 given by Eq. (9.53), the spin-orbital energies were calculated for

$\ell = 1$ using Eq. (9.70) to compare to the effect of different ℓ quantum numbers. There is

agreement between the magnitude of the predicted results given in TABLE VIII and the

20   experimental dependence on the ℓ quantum number as given in TABLE VII.

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TABLE VIII. Calculated spin-orbital energies of He I singlet excited states with $\ell = 1$ as a function of the radius of the outer electron.

n	r_2 (a_{He}) [a]	Term Symbol	$E_{s/o}$ spin-orbital coupling [b] (eV)
2	2.01873	$1s2p\ {}^1P^0$	0.0000439
3	3.01873	$1s3p\ {}^1P^0$	0.0000131
4	4.01873	$1s4p\ {}^1P^0$	0.0000056
5	5.01873	$1s5p\ {}^1P^0$	0.0000029
6	6.01873	$1s6p\ {}^1P^0$	0.0000017
7	7.01873	$1s7p\ {}^1P^0$	0.0000010

[a] Radius of the outer electron 2 from Eq. (9.53).

[b] The spin-orbital coupling energy of electron 2 from Eq. (9.70) using r_2 from Eq. (9.53).

5

4. Systems

Embodiments of the system for performing computing and rendering of the nature of excited-state atomic and atomic-ionic electrons using the physical solutions

10   may comprise a general purpose computer. Such a general purpose computer may have any number of basic configurations. For example, such a general purpose computer may comprise a central processing unit (CPU), one or more specialized processors, system memory, a mass storage device such as a magnetic disk, an optical disk, or other storage device, an input means such as a keyboard or mouse, a display device, and a printer or

15   other output device. A system implementing the present invention can also comprise a special purpose computer or other hardware system and all should be included within its scope.

The display can be static or dynamic such that spin and angular motion with corresponding momenta can be displayed in an embodiment. The displayed information

20   is useful to anticipate reactivity, physical properties, and optical absorption and emission. The insight into the nature of atomic and atomic-ionic excited-state electrons can permit the solution and display of those of other atoms and atomic ions and provide utility to anticipate their reactivity and physical properties as well as facilitate the development of light sources and materials that respond to light.

25   Embodiments within the scope of the present invention also include computer program products comprising computer readable medium having embodied therein program code means. Such computer readable media can be any available media which can be accessed by a general purpose or special purpose computer. By way of example,

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and not limitation, such computer readable media can comprise RAM, ROM, EPROM, CD ROM, DVD or other optical disk storage, magnetic disk storage or other magnetic storage devices, or any other medium which can embody the desired program code means and which can be accessed by a general purpose or special purpose computer.

5   Combinations of the above should also be included within the scope of computer readable media. Program code means comprises, for example, executable instructions and data which cause a general purpose computer or special purpose computer to perform a certain function of a group of functions.

A specific example of the rendering of the electron of atomic hydrogen using

10   Mathematica and computed on a PC is shown in FIGURE 1. The algorithm used was

To generate a spherical shell:

SphericalPlot3D[1,{q,0,p},{f,0,2p},Boxed®False,Axes®False];. The rendering can be viewed from different perspectives. A specific example of the rendering of atomic

15   hydrogen using Mathematica and computed on a PC is shown in FIGURE 1. The algorithm used was

To generate the picture of the electron and proton:

Electron=SphericalPlot3D[1,{q,0,p},{f,0,2p-p/2},Boxed®False,Axes®False];

20   Proton=Show[Graphics3D[{Blue,PointSize[0.03],Point[{0,0,0}]}],Boxed®False]; Show[Electron,Proton];

Specific examples of the rendering of the spherical-and-time-harmonic-electron-charge-density functions of non-excited and excited-state electrons using Mathematica

25   and computed on a PC are shown in FIGURE 3. The algorithm used was

To generate L1MO:

L1MOcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<.2

30   666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,RG BColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.08 5,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000

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,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],
det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£
2,RGBColor[0.674,0.079,1.000]]];

5   L1MO=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]
     Sin[phi],Cos[theta],L1MOcolors[theta,phi,1+Cos[theta]]},{theta,0,Pi},{phi,0,2Pi},Boxe
     d®False,Axes®False,Lighting®False,PlotPoints®{20,20},ViewPoint®{-0.273,-
     2.030,3.494}];

10   **To generate L1MX:**

     L1MXcolors[theta_, phi_, det_] = Which[det < 0.1333, RGBColor[1.000, 0.070,
        0.079],det < .2666, RGBColor[1.000, 0.369, 0.067],det < .4, RGBColor[1.000, 0.681,
        0.049],det < .5333, RGBColor[0.984, 1.000, 0.051], det < .6666, RGBColor[0.673,
15   1.000, 0.058], det < .8, RGBColor[0.364, 1.000, 0.055],det < .9333, RGBColor[0.071,
        1.000, 0.060], det < 1.066, RGBColor[0.085, 1.000, 0.388],det < 1.2, RGBColor[0.070,
        1.000, 0.678], det < 1.333, RGBColor[0.070, 1.000, 1.000],det < 1.466,
        RGBColor[0.067, 0.698, 1.000],   det < 1.6, RGBColor[0.075, 0.401, 1.000],det <
        1.733, RGBColor[0.067, 0.082, 1.000],   det < 1.866, RGBColor[0.326, 0.056,
20   1.000],det <= 2, RGBColor[0.674, 0.079, 1.000]]];

     L1MX=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]
     Sin[phi],Cos[theta],L1MXcolors[theta,phi,1+Sin[theta]
     Cos[phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoint
25   s®{20,20},ViewPoint®{-0.273,-2.030,3.494}];

     To generate L1MY:

     L1MYcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<.2
30   666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,RG
     BColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor[
     0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.08
     5,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000

,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],
det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£
2,RGBColor[0.674,0.079,1.000]];

5   L1MY=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]
     Sin[phi],Cos[theta],L1MYcolors[theta,phi,1+Sin[theta]
     Sin[phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoints
     ®{20,20}];

10   **To generate L2MO:**

     L2MOcolors[theta_, phi_, det_] = Which[det < 0.2, RGBColor[1.000, 0.070, 0.079],det
         < .4, RGBColor[1.000, 0.369, 0.067],det < .6, RGBColor[1.000, 0.681, 0.049],det < .8,
         RGBColor[0.984, 1.000, 0.051],det < 1, RGBColor[0.673, 1.000, 0.058],det < 1.2,
15   RGBColor[0.364, 1.000, 0.055],det < 1.4, RGBColor[0.071, 1.000, 0.060],det < 1.6,
         RGBColor[0.085, 1.000, 0.388],det < 1.8, RGBColor[0.070, 1.000, 0.678],det < 2,
         RGBColor[0.070, 1.000, 1.000],det < 2.2, RGBColor[0.067, 0.698, 1.000],det < 2.4,
         RGBColor[0.075, 0.401, 1.000],det < 2.6, RGBColor[0.067, 0.082, 1.000],det < 2.8,
         RGBColor[0.326, 0.056, 1.000],det <= 3, RGBColor[0.674, 0.079, 1.000]]];
20
     L2MO=ParametricPlot3D[{Sin[theta] Cos[phi], Sin[theta] Sin[phi], Cos[theta],
         L2MOcolors[theta, phi, 3Cos[theta] Cos[theta]]},
         {theta, 0, Pi}, {phi, 0, 2Pi},
         Boxed -> False, Axes -> False, Lighting -> False,
25   PlotPoints-> {20, 20},
         ViewPoint->{-0.273, -2.030, 3.494}];

     To generate L2MF:

30   L2MFcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<.2
     666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,RG
     BColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor[
     0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.08

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5,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000

,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],

det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£

2,RGBColor[0.674,0.079,1.000]]];

5

L2MF=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]

Sin[phi],Cos[theta],L2MFcolors[theta,phi,1+.72618 Sin[theta] Cos[phi] 5 Cos[theta]

Cos[theta]-.72618 Sin[theta]

Cos[phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoint

10   s®{20,20},ViewPoint®{-0.273,-2.030,2.494}];

To generate L2MX2Y2:

L2MX2Y2colors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],d

15   et<.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333

,RGBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBCo

lor[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.

085,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.0

00,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.00

20   0],det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],de

t£2,RGBColor[0.674,0.079,1.000]]];

L2MX2Y2=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]

Sin[phi],Cos[theta],L2MX2Y2colors[theta,phi,1+Sin[theta] Sin[theta] Cos[2

25   phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoints®{

20,20},ViewPoint®{-0.273,-2.030,3.494}];

To generate L2MXY:

30   L2MXYcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<

.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,R

GBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor

[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.08

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5,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000

,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],

det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£

2,RGBColor[0.674,0.079,1.000]]];

5

ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]

Sin[phi],Cos[theta],L2MXYcolors[theta,phi,1+Sin[theta] Sin[theta] Sin[2

phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoints®{

20,20},ViewPoint®{-0.273,-2.030,3.494}];

10

     The radii of orbitspheres of the electrons of each excited-state atom and atomic

ion are calculated by solving the force balance equation given by Maxwell's equations

for a given set of quantum numbers, and the state is displayed as modulated charge-

density waves on each two-dimensional orbitsphere at each calculated radius. A

15   computer rendering of the helium atom in the n=2, ℓ=1 excited state according to the

present Invention is shown in FIGURE 7. The algorithm used was

   <<Calculus`VectorAnalysis`

   <<Graphics`ParametricPlot3D`

20   <<Graphics`Shapes`

   <<Graphics`Animation`

   <<Graphics`SurfaceOfRevolution`

   <<Graphics`Colors`

25

   Electron=SphericalPlot3D[Evaluate[Append[{.25},{Green}]],{theta,0,Pi},{theta,

       0,2Pi-Pi/2},Boxed\[Rule]False,Axes\[Rule]False,PlotPoints\[Rule]{20,

       20},Lighting\[Rule]False];

30   Proton=Show[Graphics3D[{

       Red,PointSize[0.01],Point[{0,0,0}]}],Boxed\[Rule]False];

   InnerH=Show[Electron,Proton];

60

```
L1MXcolors[theta_, phi_, det_] =
        Which[det < 0.1333, RGBColor[1.000, 0.070, 0.079],
        det < .2666, RGBColor[1.000, 0.369, 0.067],
5       det < .4, RGBColor[1.000, 0.681, 0.049],
        det < .5333, RGBColor[0.984, 1.000, 0.051],
        det < .6666, RGBColor[0.673, 1.000, 0.058],
        det < .8, RGBColor[0.364, 1.000, 0.055],
        det < .9333, RGBColor[0.071, 1.000, 0.060],
10      det < 1.066, RGBColor[0.085, 1.000, 0.388],
        det < 1.2, RGBColor[0.070, 1.000, 0.678],
        det < 1.333, RGBColor[0.070, 1.000, 1.000],
        det < 1.466, RGBColor[0.067, 0.698, 1.000],
        det < 1.6, RGBColor[0.075, 0.401, 1.000],
15      det < 1.733, RGBColor[0.067, 0.082, 1.000],
        det < 1.866, RGBColor[0.326, 0.056, 1.000],
        det <= 2, RGBColor[0.674, 0.079, 1.000]];


   \!\(\(Do[\[IndentingNewLine]L1MX = ParametricPlot3D[
20      Evaluate[Append[\[IndentingNewLine]{Sin[theta]\ Cos[
        phi], Sin[theta]\
        Sin[phi], Cos[theta]}, \[IndentingNewLine]{
        EdgeForm[], L1MXcolors[theta, phi + \((\(\(\(2  Pi\)\)\/30\) i)\),
        1 + Sin[theta]\ Cos[phi + \((\(\(\(2
25      Pi\)\)\/30\) i)\)]]}\[IndentingNewLine]]], {theta, 0, Pi}, {phi,
        0, Pi/2 + Pi}, Boxed \[Rule] False, Axes \[Rule]
        False, Lighting \[Rule] False,
        PlotPoints \[Rule] {35, 35}, ViewPoint \[Rule] {
        0, 0, 2},
30      ImageSize \[Rule] 72*6], \[IndentingNewLine]{i,
        1, 1}];\)\)


   Show[InnerH,L1MX,Lighting\[Rule]False];
```

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The present Invention may be embodied in other specific forms without departing from the spirit or essential attributes thereof and, accordingly, reference should be made to the appended claims, rather than to the foregoing specification, as indicating the scope

5   of the Invention.

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         Mechanical%20Solutions%20for%20One-%20Through%20Twenty-

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I claim:

1. A system of computing and rendering the nature of excited-state atomic and atomic ionic electrons comprising:

   physical, Maxwellian solutions of the charge, mass, and current density functions of atoms and atomic ions,

   a processing means, and

   an output means.

2. The system of claim 1, further comprising a display in communication with the output means.

3. The system of claim 2, wherein the display is at least one of visual or graphical media.

4. The system of claim 2, wherein the display is at least one of static or dynamic.

5. The system of claim 4, wherein at least one of spin and orbital angular motion is be displayed.

6. The system of claim 2, wherein the displayed information is used to model at least one of reactivity, physical properties, optical absorption and optical emission.

7. The system of claim 2, wherein the displayed information is used to model other of excited-state atoms and atomic ions and provide utility to anticipate their reactivity, physical properties, optical absorption and optical emission.

8. The system of claim 1, wherein the processing means is a general purpose computer.

9. The system of claim 8, wherein the general purpose computer comprises a central processing unit (CPU), one or more specialized processors, system memory, a mass storage device such as a magnetic disk, an optical disk, or other storage device, an input means such as a keyboard or mouse, a display device, and a printer or other output device.

10. The system of claim 1, wherein the processing means comprises a special purpose computer or other hardware system.

5   11. The system of claim 1, further comprising computer program products.

12. The system of claim 11, comprising computer readable medium having embodied therein program code means.

10   13. The system of claim 12, wherein the computer readable media is any available media which can be accessed by a general purpose or special purpose computer.

14. The system of claim 13, wherein the computer readable media comprises at least one of RAM, ROM, EPROM, CD ROM, DVD or other optical disk storage, magnetic

15   disk storage or other magnetic storage devices, or any other medium which can embody the desired program code means and which can be accessed by a general purpose or special purpose computer.

15. The system of claim 14, wherein the program code means comprises executable

20   instructions and data which cause a general purpose computer or special purpose computer to perform a certain function of a group of functions.

16. The system of claim 15, wherein the program code is Mathematica programmed with an algorithm based on the physical solutions, and the computer is a PC.

25

17. The system of claim 16, wherein the algorithm for the rendering of the electron of atomic hydrogen using Mathematica and computed on a PC is

SphericalPlot3D[1,{q,0,p},{f,0,2p},Boxed®False,Axes®False];

30   and the algorithm for the rendering of atomic hydrogen using Mathematica and computed on a PC is

Electron=SphericalPlot3D[1,{q,0,p},{f,0,2p-p/2},Boxed®False,Axes®False];

Proton=Show[Graphics3D[{Blue,PointSize[0.03],Point[{0,0,0}]}],Boxed®False];

Show[Electron,Proton].

5 **18.** The system of claim 16, wherein the algorithm for the rendering of the spherical-
and-time-harmonic-electron-charge-density functions using Mathematica and
computed on a PC are

To generate L1MO:

10 L1MOcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<.2
666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,RG
BColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor[
0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.08
5,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000
15 ,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],
det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£
2,RGBColor[0.674,0.079,1.000]]];

L1MO=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]
20 Sin[phi],Cos[theta],L1MOcolors[theta,phi,1+Cos[theta]]},{theta,0,Pi},{phi,0,2Pi},Boxe
d®False,Axes®False,Lighting®False,PlotPoints®{20,20},ViewPoint®{-0.273,-
2.030,3.494}];

To generate L1MX:

25

L1MXcolors[theta_, phi_, det_] = Which[det < 0.1333, RGBColor[1.000, 0.070,
0.079],det < .2666, RGBColor[1.000, 0.369, 0.067],det < .4, RGBColor[1.000, 0.681,
0.049],det < .5333, RGBColor[0.984, 1.000, 0.051], det < .6666, RGBColor[0.673,
1.000, 0.058], det < .8, RGBColor[0.364, 1.000, 0.055],det < .9333, RGBColor[0.071,
30 1.000, 0.060], det < 1.066, RGBColor[0.085, 1.000, 0.388],det < 1.2, RGBColor[0.070,
1.000, 0.678], det < 1.333, RGBColor[0.070, 1.000, 1.000],det < 1.466,
RGBColor[0.067, 0.698, 1.000],   det < 1.6, RGBColor[0.075, 0.401, 1.000],det <
1.733, RGBColor[0.067, 0.082, 1.000],   det < 1.866, RGBColor[0.326, 0.056,

1.000],det <= 2, RGBColor[0.674, 0.079, 1.000]];

L1MX=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]

Sin[phi],Cos[theta],L1MXcolors[theta,phi,1+Sin[theta]

5   Cos[phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoint

s®{20,20},ViewPoint®{-0.273,-2.030,3.494}];

To generate L1MY:

10   L1MYcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<.2

666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,RG

BColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor[

0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.08

5,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000

15   ,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],

det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£

2,RGBColor[0.674,0.079,1.000]];

L1MY=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]

20   Sin[phi],Cos[theta],L1MYcolors[theta,phi,1+Sin[theta]

Sin[phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoints

®{20,20}];

To generate L2MO:

25

L2MOcolors[theta_, phi_, det_] = Which[det < 0.2, RGBColor[1.000, 0.070, 0.079],det

< .4, RGBColor[1.000, 0.369, 0.067],det < .6, RGBColor[1.000, 0.681, 0.049],det < .8,

RGBColor[0.984, 1.000, 0.051],det < 1, RGBColor[0.673, 1.000, 0.058],det < 1.2,

RGBColor[0.364, 1.000, 0.055],det < 1.4, RGBColor[0.071, 1.000, 0.060],det < 1.6,

30   RGBColor[0.085, 1.000, 0.388],det < 1.8, RGBColor[0.070, 1.000, 0.678],det < 2,

RGBColor[0.070, 1.000, 1.000],det < 2.2, RGBColor[0.067, 0.698, 1.000],det < 2.4,

RGBColor[0.075, 0.401, 1.000],det < 2.6, RGBColor[0.067, 0.082, 1.000],det < 2.8,

RGBColor[0.326, 0.056, 1.000],det <= 3, RGBColor[0.674, 0.079, 1.000]];

L2MO=ParametricPlot3D[{Sin[theta] Cos[phi], Sin[theta] Sin[phi], Cos[theta],

L2MOcolors[theta, phi, 3Cos[theta] Cos[theta]]},

{theta, 0, Pi}, {phi, 0, 2Pi},

5   Boxed -> False, Axes -> False, Lighting -> False,

PlotPoints-> {20, 20},

ViewPoint->{-0.273, -2.030, 3.494}];

To generate L2MF:

10

L2MFcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<.2
666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,RG
BColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor[
0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.08
15   5,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000
,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],
det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£
2,RGBColor[0.674,0.079,1.000]]];

20   L2MF=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]
Sin[phi],Cos[theta],L2MFcolors[theta,phi,1+.72618 Sin[theta] Cos[phi] 5 Cos[theta]
Cos[theta]-.72618 Sin[theta]
Cos[phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoint
s®{20,20},ViewPoint®{-0.273,-2.030,2.494}];

25

To generate L2MX2Y2:

L2MX2Y2colors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],d
et<.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333
30   ,RGBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBCo
lor[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.
085,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.0
00,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.00

0],det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],de
t£2,RGBColor[0.674,0.079,1.000]]];

L2MX2Y2=ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]

5   Sin[phi],Cos[theta],L2MX2Y2colors[theta,phi,1+Sin[theta] Sin[theta] Cos[2

phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoints®{

20,20},ViewPoint®{-0.273,-2.030,3.494}];

To generate L2MXY:

10

L2MXYcolors[theta_,phi_,det_]=Which[det<0.1333,RGBColor[1.000,0.070,0.079],det<

.2666,RGBColor[1.000,0.369,0.067],det<.4,RGBColor[1.000,0.681,0.049],det<.5333,R

GBColor[0.984,1.000,0.051],det<.6666,RGBColor[0.673,1.000,0.058],det<.8,RGBColor

[0.364,1.000,0.055],det<.9333,RGBColor[0.071,1.000,0.060],det<1.066,RGBColor[0.08

15   5,1.000,0.388],det<1.2,RGBColor[0.070,1.000,0.678],det<1.333,RGBColor[0.070,1.000

,1.000],det<1.466,RGBColor[0.067,0.698,1.000],det<1.6,RGBColor[0.075,0.401,1.000],

det<1.733,RGBColor[0.067,0.082,1.000],det<1.866,RGBColor[0.326,0.056,1.000],det£

2,RGBColor[0.674,0.079,1.000]]];

20   ParametricPlot3D[{Sin[theta] Cos[phi],Sin[theta]

Sin[phi],Cos[theta],L2MXYcolors[theta,phi,1+Sin[theta] Sin[theta] Sin[2

phi]]},{theta,0,Pi},{phi,0,2Pi},Boxed®False,Axes®False,Lighting®False,PlotPoints®{

20,20},ViewPoint®{-0.273,-2.030,3.494}].

25   **19.** The system of claim 16, wherein the algorithm for the rendering of the spherical-
and-time-harmonic-electron-charge-density functions using Mathematica and
computed on a PC for the helium atom in the n=2, ℓ =1 excited state is

<<Calculus`VectorAnalysis`

30   <<Graphics`ParametricPlot3D`

<<Graphics`Shapes`

<<Graphics`Animation`

<<Graphics`SurfaceOfRevolution`

```
<<Graphics`Colors`


Electron=SphericalPlot3D[Evaluate[Append[{.25},{Green}]],{theta,0,Pi},{theta,
        0,2Pi-Pi/2},Boxed\[Rule]False,Axes\[Rule]False,PlotPoints\[Rule]{20,
        20},Lighting\[Rule]False];


Proton=Show[Graphics3D[{
        Red,PointSize[0.01],Point[{0,0,0}]}],Boxed\[Rule]False];


InnerH=Show[Electron,Proton];


L1MXcolors[theta_, phi_, det_] =
        Which[det < 0.1333, RGBColor[1.000, 0.070, 0.079],
        det < .2666, RGBColor[1.000, 0.369, 0.067],
        det < .4, RGBColor[1.000, 0.681, 0.049],
        det < .5333, RGBColor[0.984, 1.000, 0.051],
        det < .6666, RGBColor[0.673, 1.000, 0.058],
        det < .8, RGBColor[0.364, 1.000, 0.055],
        det < .9333, RGBColor[0.071, 1.000, 0.060],
        det < 1.066, RGBColor[0.085, 1.000, 0.388],
        det < 1.2, RGBColor[0.070, 1.000, 0.678],
        det < 1.333, RGBColor[0.070, 1.000, 1.000],
        det < 1.466, RGBColor[0.067, 0.698, 1.000],
        det < 1.6, RGBColor[0.075, 0.401, 1.000],
        det < 1.733, RGBColor[0.067, 0.082, 1.000],
        det < 1.866, RGBColor[0.326, 0.056, 1.000],
        det <= 2, RGBColor[0.674, 0.079, 1.000]]];


\!\(\(Do[\[IndentingNewLine]L1MX = ParametricPlot3D[
        Evaluate[Append[\[IndentingNewLine]{Sin[theta]\ Cos[
          phi], Sin[theta]\
            Sin[phi], Cos[theta]}, \[IndentingNewLine]{
              EdgeForm[], L1MXcolors[theta, phi + \((\(\(\(2  Pi\)\)\/30\) i)\),
```

1 + Sin[theta]\ Cos[phi + \((\(\(2

Pi\)\/30\) i)\)]]}\[IndentingNewLine]]], {theta, 0, Pi}, {phi,

0, Pi/2 + Pi}, Boxed \[Rule] False, Axes \[Rule]

False, Lighting \[Rule] False,

5   PlotPoints \[Rule] {35, 35}, ViewPoint \[Rule] {

0, 0, 2},

ImageSize \[Rule] 72*6], \[IndentingNewLine]{i,

1, 1}];\)\)\)

10   Show[InnerH,L1MX,Lighting\[Rule]False].

20. The system of claim 1, wherein the physical, Maxwellian solutions of the charge, mass, and current density functions of excited-state atoms and atomic ions comprises a solution of the classical wave equation $\left[\nabla^2 - \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \right] \rho(r,\theta,\phi,t) = 0$.

15

21. The system of claim 20, wherein the time, radial, and angular solutions of the wave equation are separable.

22. The system of claim 21, wherein the radii of orbitspheres of the electrons of each
20   excited-state atom and atomic ion are calculated by solving the force balance equation given by Maxwell's equation for a given set of quantum numbers, and the state is displayed as modulated charge-density waves on each two-dimensional orbitsphere at each calculated radius.

25   23. The system of claim 21, wherein radial function which does satisfy the boundary conditions is a radial delta function

$$f(r) = \frac{1}{r^2} \delta(r - r_n).$$

24. The system of claim 23, wherein the boundary condition is met for a time harmonic
30   function when the relationship between an allowed radius and the electron wavelength is given by

$$2\pi r_n = \lambda_n,$$

$$\omega = \frac{\hbar}{m_e r^2}, \text{ and}$$

$$v = \frac{\hbar}{m_e r}$$

where ω is the angular velocity of each point on the electron surface, v is the velocity of each point on the electron surface, and r is the radius of the electron.

25. The system of claim 24, wherein the spin function is given by the uniform function $Y_0^{\ 0}(\phi, \theta)$ comprising angular momentum components of $\mathbf{L}_{xy} = \dfrac{\hbar}{4}$ and $\mathbf{L}_z = \dfrac{\hbar}{2}$.

26. The system of claim 25, wherein the atomic and atomic ionic charge and current density functions of excited-state electrons are described by a charge-density (mass-density) function which is the product of a radial delta function, two angular functions (spherical harmonic functions), and a time harmonic function:

$$\rho(r,\theta,\phi,t) = f(r)A(\theta,\phi,t) = \frac{1}{r^2}\delta(r - r_n)A(\theta,\phi,t); \qquad A(\theta,\phi,t) = Y(\theta,\phi)k(t)$$

wherein the spherical harmonic functions correspond to a traveling charge density wave confined to the spherical shell which gives rise to the phenomenon of orbital angular momentum.

27. The system of claim 26, wherein based on the radial solution, the angular charge and current-density functions of the electron, $A(\theta, \phi, t)$, must be a solution of the wave equation in two dimensions (plus time),

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]A(\theta,\phi,t) = 0$$

where $\rho(r, \theta, \phi, t) = f(r)A(\theta,\phi,t) = \frac{1}{r^2}\delta(r - r_n)A(\theta,\phi,t)$ and $A(\theta, \phi, t) = Y(\theta,\phi)k(t)$

$$\left[\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)_{r,\phi} + \frac{1}{r^2 \sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}\right)_{r,\theta} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]A(\theta,\phi,t) = 0$$

where v is the linear velocity of the electron.

28. The system of claim 27, wherein the charge-density functions including the time-function factor are

$$\ell = 0$$

5

$$\rho(r,\theta,\phi,t) = \frac{e}{8\pi r^2}[\delta(r - r_n)]\left[Y_0^0(\theta,\phi) + Y_\ell^m(\theta,\phi)\right]$$

$$\ell \neq 0$$

10

$$\rho(r,\theta,\phi,t) = \frac{e}{4\pi r^2}[\delta(r - r_n)]\left[Y_0^0(\theta,\phi) + \text{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\}\right]$$

where $Y_\ell^m(\theta,\phi)$ are the spherical harmonic functions that spin about the z-axis with

angular frequency ω_n with $Y_0^0(\theta,\phi)$ the constant function;

$$\text{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\} = P_\ell^m(\cos\theta)\cos\left(m\phi + \omega_n' t\right) \text{ where to keep the form of the}$$

15   spherical harmonic as a traveling wave about the z-axis, $\omega_n' = m\omega_n$.

29. The system of claim 28, wherein the spin and angular moment of inertia, I, angular

momentum, L, and energy, E, for quantum number ℓ are given by

$$\ell = 0$$

20

$$I_z = I_{spin} = \frac{m_e r_n^2}{2}$$

$$L_z = I\omega\mathbf{i}_z = \pm\frac{\hbar}{2}$$

$$E_{rotational} = E_{rotational,\,spin} = \frac{1}{2}\left[I_{spin}\left(\frac{\hbar}{m_e r_n^2}\right)^2\right] = \frac{1}{2}\left[\frac{m_e r_n^2}{2}\left(\frac{\hbar}{m_e r_n^2}\right)^2\right] = \frac{1}{4}\left[\frac{\hbar^2}{2I_{spin}}\right]$$

25        $$\ell \neq 0$$

$$I_{orbital} = m_e r_n^2\left[\frac{\ell(\ell+1)}{\ell^2 + \ell + 1}\right]^{\frac{1}{2}}$$

$$L_z = m\hbar$$

$$L_{z\,total} = L_{z\,spin} + L_{z\,orbital}$$

$$E_{rotational,\,orbital} = \frac{\hbar^2}{2I}\left[\frac{\ell(\ell+1)}{\ell^2 + 2\ell + 1}\right]$$

$$T = \frac{\hbar^2}{2m_e r_n^2}$$

$$\langle E_{rotational,\,orbital}\rangle = 0.$$

30. The system of claim 1, wherein the initial force balance equation for one-electron atoms and ions before excitation is

$$\frac{m_e}{4\pi r_1^2}\frac{v_1^2}{r_1} = \frac{e}{4\pi r_1^2}\frac{Ze}{4\pi\varepsilon_o r_1^2} - \frac{1}{4\pi r_1^2}\frac{\hbar^2}{m_p r_n^3}$$

$$r_1 = \frac{a_H}{Z}$$

where a_H is the radius of the hydrogen atom.

31. The system of claim 30, wherein from Maxwell's equations, the potential energy V, kinetic energy T, electric energy or binding energy E_{ele} are

$$V = \frac{-Ze^2}{4\pi\varepsilon_o r_1} = \frac{-Z^2 e^2}{4\pi\varepsilon_o a_H} = -Z^2 \, X \, 4.3675 \, X \, 10^{-18} \, J = -Z^2 \, X \, 27.2 \; eV$$

$$T = \frac{Z^2 e^2}{8\pi\varepsilon_o a_H} = Z^2 \, X \, 13.59 \; eV$$

$$T = E_{ele} = -\frac{1}{2}\varepsilon_o \int_\infty^{r_1}\mathbf{E}^2 dv \quad \text{where} \quad \mathbf{E} = -\frac{Ze}{4\pi\varepsilon_o r^2}$$

$$E_{ele} = -\frac{Z^2 e^2}{8\pi\varepsilon_o a_H} = -Z^2 \, X \, 2.1786 \, X \, 10^{-18} \; J = -Z^2 \, X \, 13.598 \; eV.$$

32. The system of claim 1, wherein the initial force balance equation solution before excitation of two-electron atoms is a central force balance equation with the nonradiation condition given by

$$\frac{m_e}{4\pi r_2^2}\frac{v_2^2}{r_2} = \frac{e}{4\pi r_2^2}\frac{(Z-1)e}{4\pi\varepsilon_o r_2^2} + \frac{1}{4\pi r_2^2}\frac{\hbar^2}{Zm_e r_2^3}\sqrt{s(s+1)}$$

which gives the radius of both electrons as

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$$r_2 = r_1 = a_0 \left(\frac{1}{Z-1} - \frac{\sqrt{s(s+1)}}{Z(Z-1)} \right); \ s = \frac{1}{2}.$$

33. The system of claim 32, wherein the ionization energy for helium, which has no electric field beyond r_1 is given by

$$Ionization \ Energy(He) = -E(electric) + E(magnetic)$$

where,

$$E(electric) = -\frac{(Z-1)e^2}{8\pi\varepsilon_o r_1}$$

$$E(magnetic) = \frac{2\pi\mu_0 e^2 \hbar^2}{m_e^2 r_1^3}$$

For $3 \le Z$

$$Ionization \ Energy = -Electric \ Energy - \frac{1}{Z} Magnetic \ Energy.$$

34. The system of claim 1, wherein the electrons of excited states of one and multielectron atoms all exist as orbitspheres of discrete radii which are given by r_n of the radial Dirac delta function, $\delta(r - r_n)$.

35. The system of claim 34, wherein the electrons of excited states of one and multielectron atoms all exist as orbitspheres of discrete radii which are given by r_n of the radial Dirac delta function, $\delta(r - r_n)$ that serve as resonator cavities and trap electromagnetic radiation of discrete resonant frequencies.

36. The system of claim 35, wherein photon absorption occurs as an excitation of a resonator mode.

37. The system of claim 36, wherein the free space photon also comprises a radial Dirac delta function, and the angular momentum of the photon given by

$$\mathbf{m} = \int \frac{1}{8\pi c} Re[\mathbf{r} \times (\mathbf{E} \times \mathbf{B}^*)] dx^4 = \hbar \ \text{is conserved for the solutions for the resonant}$$

photons and excited state electron functions.

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38. The system of claim 37, wherein the change in angular frequency of the electron is equal to the angular frequency of the resonant photon that excites the resonator cavity mode corresponding to the transition, and the energy is given by Planck's equation.

39. The system of claim 38, wherein for each multipole state with a single m value the relationship between the angular momentum M_z, energy U, and angular frequency ω is given by:

$$\frac{dM_z}{dr} = \frac{m}{\omega}\frac{dU}{dr}$$

independent of r where m is an integer such that the ratio of the square of the angular momentum, M^2, to the square of the energy, U^2, for a pure (ℓ, m) multipole is given by

$$\frac{M^2}{U^2} = \frac{m^2}{\omega^2}.$$

40. The system of claim 39, wherein the radiation from such a multipole of order (ℓ, m) carries off $m\hbar$ units of the z component of angular momentum per photon of energy $\hbar\omega$.

41. The system of claim 40, wherein the photon and the electron can each posses only $\hbar$ of angular momentum which requires that the radiation field contain m photons.

42. The system of claim 41, wherein during excitation the spin, orbital, or total angular momentum of the orbitsphere can change by zero or $\pm$ $\hbar$.

43. The system of claim 42, wherein the selection rules for multipole transitions between quantum states arise from conservation of the photon's multipole moment and angular momentum of $\hbar$.

44. The system of claim 43, wherein in an excited state, the time-averaged mechanical angular momentum and rotational energy associated with the traveling charge-density wave on the orbitsphere is zero, and the angular momentum of $\hbar$ of the photon that excites the electronic state is carried by the fields of the trapped photon.

45. The system of claim 44, wherein the amplitudes of the rotational moment of inertia, angular momentum, and energy that couple to external magnetic and electromagnetic fields are given

$$\text{by } I_{orbital} = m_e r_n^2 \left[\frac{\ell(\ell+1)}{\ell^2+2\ell+1} \right]^{\frac{1}{2}} = m_e r_n^2 \sqrt{\frac{\ell}{\ell+1}} \, ,$$

$$\mathbf{L} = I\omega \mathbf{i}_z = I_{orbital}\omega \mathbf{i}_z = m_e r_n^2 \left[\frac{\ell(\ell+1)}{\ell^2+2\ell+1} \right]^{\frac{1}{2}} \omega \mathbf{i}_z = m_e r_n^2 \frac{\hbar}{m_e r_n^2} \sqrt{\frac{\ell}{\ell+1}} = \hbar \sqrt{\frac{\ell}{\ell+1}} \, , \text{ and}$$

$$E_{rotational \text{ orbital}} = \frac{\hbar^2}{2I} \left[\frac{\ell(\ell+1)}{\ell^2+2\ell+1} \right] = \frac{\hbar^2}{2I} \left[\frac{\ell}{\ell+1} \right] = \frac{\hbar^2}{2m_e r_n^2} \left[\frac{\ell}{\ell+1} \right] , \text{ respectively.}$$

46. The system of claim 45, wherein the electron charge-density waves are nonradiative due to the angular motion, but excited states are radiative due to a radial dipole that arises from the presence of the trapped photon.

47. The system of claim 46, wherein the total number of multipoles, $N_{\ell,s}$, of an energy level corresponding to a principal quantum number n where each multipole corresponds to an ℓ and m $_\ell$ quantum number is

$$N_{\ell,s} = \sum_{\ell=0}^{n-1} \sum_{m_\ell=-\ell}^{+\ell} 1 = \sum_{\ell=0}^{n-1} 2\ell + 1 = (\ell+1)^2 = \ell^2 + 2\ell + 1 = n^2 .$$

48. The system of claim 47, wherein the photon's electric field superposes that of the nucleus for $r_1 < r < r_2$ such that the radial electric field has a magnitude proportional to e/n at the electron 2 (the excited electron) where $n = 2,3,4,...$ for excited states such that U is decreased by the factor of $1/n^2$.

49. The system of claim 48, wherein the "trapped photon" of the excited state is a "standing electromagnetic wave" which actually is a traveling wave that propagates on the surface around the z-axis, and its source current is only at the orbitsphere.

50. The system of claim 49, wherein the time-function factor, $k(t)$, for the "standing wave" is identical to the time-function factor of the orbitsphere in order to satisfy the boundary (phase) condition at the orbitsphere surface such that the angular frequency

of the "trapped photon" is identical to the angular frequency of the electron orbitsphere, ω_n.

51. The system of claim 50, wherein the angular functions of the "trapped photon" are
5   identical to the spherical harmonic angular functions of the electron orbitsphere.

52. The system of claim 51, wherein combining $k(t)$ with the ϕ-function factor of the
        spherical harmonic gives $e^{i(m\phi - \omega_n t)}$ for both the electron and the "trapped photon"
        function.
10

53. The system of claim 52, wherein the photon "standing wave" in an excited electronic
        state is a solution of Laplace's equation in spherical coordinates with source currents
        matching those of the electron orbitsphere "glued" to the electron and phase-locked
        to the electron current density wave that travel on the surface with a radial electric
15   field.

54. The system of claim 53, wherein the photon field is purely radial since the field is
        traveling azimuthally at the speed of light even though the spherical harmonic
        function has a velocity less than light speed.
20

55. The system of claim 54, wherein the photon field does not change the nature of the
        electrostatic field of the nucleus or its energy except at the position of the electron.

56. The system of claim 55, wherein the photon "standing wave" function comprises a
25   radial Dirac delta function that "samples" the Laplace equation solution only at the
        position infinitesimally inside of the electron current-density function and
        superimposes with the proton field to give a field of radial magnitude corresponding
        to a charge of e/n where $n = 2,3,4,....$.

30   57. The system of claim 56, wherein the electric field of the nucleus for $r_1 < r < r_2$ is

$$\mathbf{E}_{nucleus} = \frac{e}{4\pi\varepsilon_o r^2} .$$

58. The system of claim 57, wherein the equation of the electric field of the "trapped photon" for $r = r_2$ where r_2 is the radius of electron 2, is

$$\mathbf{E}_{r\,photon\,n,l,m}\big|_{r=r_2} = \frac{e}{4\pi\varepsilon_o r_2^2}\left[-1+\frac{1}{n}\left[Y_0^0(\theta,\phi)+\mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_{n,t}}\right\}\right]\right]\delta(r-r_n)$$

$$\omega_n = 0 \qquad \text{for } m = 0.$$

$$\omega_n = 0$$

59. The system of claim 58, wherein the total central field for $r = r_2$ is given by the sum of the electric field of the nucleus and the electric field of the "trapped photon":

$$\mathbf{E}_{total} = \mathbf{E}_{nucleus} + \mathbf{E}_{photon}.$$

60. The system of claim 59, wherein for $r_1 < r < r_2$,

$$\mathbf{E}_{r total} = \frac{e}{4\pi\varepsilon_o r_1^2} + \frac{e}{4\pi\varepsilon_o r_2^2}\left[-1+\frac{1}{n}\left[Y_0^0(\theta,\phi)+\mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_{n,t}}\right\}\right]\right]\delta(r-r_n)$$

$$= \frac{1}{n}\frac{e}{4\pi\varepsilon_o r_2^2}\left[Y_0^0(\theta,\phi)+\mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_{n,t}}\right\}\right]\delta(r-r_n)$$

$$\omega_n = 0 \text{ for } m = 0.$$

61. The system of claim 60, wherein for $r = r_2$ and $m = 0$, the total radial electric field is

$$\mathbf{E}_{r total} = \frac{1}{n}\frac{e}{4\pi\varepsilon_o r^2}.$$

62. The system of claim 61, wherein the result is the same for the excited states of the one-electron atom in that the total radial electric field is

$$\mathbf{E}_{r total} = \frac{1}{n}\frac{e}{4\pi\varepsilon_o r^2}.$$

63. The system of claim 61, wherein for $r_1 < r < r_2$,

$$\mathbf{E}_{r total} = \frac{e}{4\pi\varepsilon_o r_1^2} + \frac{e}{4\pi\varepsilon_o r_2^2}\left[-1+\frac{1}{n}\left[Y_0^0(\theta,\phi)+\mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_t}\right\}\right]\right]\delta(r-r_n)$$

$$= \frac{1}{n}\frac{e}{4\pi\varepsilon_o r_2^2}\left[Y_0^0(\theta,\phi)+\mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_t}\right\}\right]\delta(r-r_n)$$

$$\omega_n = 0 \text{ for } m = 0$$

64. The system of claim 63, wherein the radii of the excited-state electron is determined from the force balance of the electric, magnetic, and centrifugal forces that corresponds to the minimum of energy of the system.

5

65. The system of claim 64, wherein the excited-state energies are given by the electric energies at these radii.

66. The system of claim 65, wherein electron orbitspheres may be spin paired or
10   unpaired depending on the force balance which applies to each electron wherein the electron configuration is a minimum of energy.

67. The system of claim 66, wherein the minimum energy configurations are given by solutions to Laplace's equation.

15

68. The system of claim 67, wherein the corresponding force balance of the central Coulombic, paramagnetic, and diamagnetic forces is derived for each n-electron atom that is solved for the radius of each electron.

20 69. The system of claim 68, wherein the central Coulombic force is that of a point charge at the origin since the electron charge-density functions are spherically symmetrical with a harmonic time dependence.

70. The system of claim 69, wherein the ionization energy of each electron is obtained
25   using the calculated radii in the determination of the Coulombic and any magnetic energies.

71. The system of claim 69, wherein for the singlet-excited state with $\ell = 0$, the electron source current in the excited state is a constant function that spins as a globe about
30   the z-axis:

$$\rho(r,\theta,\phi,t) = \frac{e}{8\pi r^2}[\delta(r - r_n)][Y_0^0(\theta,\phi) + Y_\ell^m(\theta,\phi)].$$

72. The system of claim 70, wherein the balance between the centrifugal and electric and magnetic forces is given by:

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} + \frac{2}{3}\frac{1}{n}\frac{\hbar^2}{2m_e r_2^3}\sqrt{s(s+1)} \quad ; \quad s = \frac{1}{2}.$$

5   **73.** The system of claim 71, wherein

$$r_2 = \left[n - \frac{\sqrt{\frac{3}{4}}}{3} \right] a_{He} \qquad n = 2,3,4,\ldots$$

74. The system of claim 72, wherein the excited-state energy is the energy stored in the electric field, E_{ele}, which is the energy of the excited-state electron (electron 2)

10   relative to the ionized electron at rest having zero energy:

$$E_{ele} = -\frac{1}{n}\frac{e^2}{8\pi\varepsilon_o r_2}.$$

75. The system of claim 73, wherein the forces on electron 2 due to the nucleus and electron 1 are radial/central, invariant of r_1, and independent of r_1 with the condition

15   that $r_1 < r_2$, such that r_2 can be determined without knowledge of r_1.

76. The system of claim 74, wherein r_1 is solved using the equal and opposite magnetic force of electron 2 on electron 1 at the radius r_2 determined from the force balance equation for electron 2 and the central Coulombic force corresponding to the charge

20   of the nucleus.

77. The system of claim 75, wherein the force balance between the centrifugal and electric and magnetic forces is

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} - \frac{1}{3n}\frac{\hbar^2}{m_e r_2^3}\sqrt{s(s+1)} \quad ; \quad s = \frac{1}{2}.$$

25   such that with $s = \frac{1}{2}$,

$$r_1^3 - \left(\frac{12n}{\sqrt{3}}r_2^3\right)r_1 + \frac{6n}{\sqrt{3}}r_2^3 = 0 \qquad n = 2,3,4,\ldots$$

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$$r_1 = r_{13} = -\sqrt{\frac{2}{3}} g \left(\cos\frac{\theta}{3} - \sqrt{3}\sin\frac{\theta}{3} \right)$$

where r_1 and r_2 are in units of a_{He}.

78. The system of claim 76, wherein for the triplet-excited state with $\ell \neq 0$, time-independent charge-density waves corresponding to the source currents travel on the surface of the orbitsphere of electron 2 about the z-axis.

79. The system of claim 77, wherein in the triplet state, the spin-spin force is paramagnetic and twice that of the singlet state.

80. The system of claim 78, wherein the force balance between the centrifugal and electric and magnetic forces is:

$$\frac{m_e v_2^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} + 2\frac{2}{3}\frac{1}{n}\frac{\hbar^2}{2m_e r_2^3}\sqrt{s(s+1)}.$$

81. The system of claim 79, wherein

$$r_2 = \left[n - \frac{2\sqrt{\frac{3}{4}}}{3} \right] a_{He} \qquad n = 2,3,4,\ldots$$

82. The system of claim 80, wherein the excited-state energy is the energy stored in the electric field, E_{ele}, which is the energy of the excited-state electron (electron 2) relative to the ionized electron at rest having zero energy:

$$E_{ele} = -\frac{1}{n}\frac{e^2}{8\pi\varepsilon_o r_2}.$$

83. The system of claim 81, wherein using r_2, r_1 is be solved using the equal and opposite magnetic force of electron 2 on electron 1 and the central Coulombic force corresponding to the nuclear charge such that the force balance between the centrifugal and electric and magnetic forces is

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$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} - \frac{2}{3n}\frac{\hbar^2}{m_e r_2^3}\sqrt{s(s+1)}$$

and with $s = \dfrac{1}{2}$,

$$r_1^3 - \left(\frac{6n}{\sqrt{3}}r_2^3\right)r_1 + \frac{3n}{\sqrt{3}}r_2^3 = 0 \qquad n = 2,3,4,...$$

$$r_1 = r_{13} = -\sqrt{\frac{2}{3}}g\left(\cos\frac{\theta}{3} - \sqrt{3}\sin\frac{\theta}{3}\right)$$

5   where r_1 and r_2 are in units of a_{He}.

84.   The system of claim 82, wherein for the singlet-excited state with $\ell \neq 0$, the electron
      source current in the excited state is the sum of constant and time-dependent
      functions where the latter, given by

10    $$\rho(r,\theta,\phi,t) = \frac{e}{4\pi r^2}[\delta(r-r_n)]\left[Y_0^0(\theta,\phi) + \mathrm{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\}\right]$$ that travels about the z-

      axis.

85.   The system of claim 83, wherein the current due to the time dependent term
      corresponding to p, d, f, etc. orbitals is

      $$\mathbf{J} = \frac{\omega_n}{2\pi}\frac{e}{4\pi r_n^2}N[\delta(r-r_n)]\mathrm{Re}\left\{Y_\ell^m(\theta,\phi)\right\}[\mathbf{u}(t)\times\mathbf{r}]$$

15    $$= \frac{\omega_n}{2\pi}\frac{e}{4\pi r_n^2}N'[\delta(r-r_n)]\left(P_\ell^m(\cos\theta)\cos(m\phi + \omega_n't)\right)[\mathbf{u}\times\mathbf{r}]$$

      $$= \frac{\omega_n}{2\pi}\frac{e}{4\pi r_n^2}N'[\delta(r-r_n)]\left(P_\ell^m(\cos\theta)\cos(m\phi + \omega_n't)\right)\sin\theta\hat{\phi}$$

      where to keep the form of the spherical harmonic as a traveling wave about the z-axis,

      $\omega_n' = m\omega_n$ and N and N' are normalization constants; the vectors are defined as

      $$\hat{\phi} = \frac{\hat{u}\times\hat{r}}{|\hat{u}\times\hat{r}|} = \frac{\hat{u}\times\hat{r}}{\sin\theta}; \qquad \hat{u} = \hat{z} = orbital\ axis$$

      $$\hat{\theta} = \hat{\phi}\times\hat{r}$$

20   "^" denotes the unit vectors $\hat{u} \equiv \dfrac{\mathbf{u}}{|\mathbf{u}|}$, non-unit vectors are designed in bold, and the

      current function is normalized.

86.   The system of claim 84, wherein the general multipole field solution to Maxwell's

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equations in a source-free region of empty space with the assumption of a time dependence $e^{i\omega_n t}$ (cgs units) is

$$\mathbf{B} = \sum_{\ell,m} \left[a_E(\ell,m) f_\ell(kr) \mathbf{X}_{\ell.m} - \frac{i}{k} a_M(\ell,m) \nabla \times g_\ell(kr) \mathbf{X}_{\ell.m} \right]$$

$$\mathbf{E} = \sum_{\ell,m} \left[\frac{i}{k} a_E(\ell,m) \nabla \times f_\ell(kr) \mathbf{X}_{\ell.m} + a_M(\ell,m) g_\ell(kr) \mathbf{X}_{\ell.m} \right];$$

the radial functions $f_\ell(kr)$ and $g_\ell(kr)$ are of the form:

$$g_\ell(kr) = A_\ell^{(1)} h_\ell^{(1)} + A_\ell^{(2)} h_\ell^{(2)}, \text{ and}$$

$\mathbf{X}_{\ell.m}$ is the vector spherical harmonic defined by

$$\mathbf{X}_{\ell,m}(\theta,\phi) = \frac{1}{\sqrt{\ell(\ell+1)}} \mathbf{L} Y_{\ell,m}(\theta,\phi) \text{ where}$$

$$\mathbf{L} = \frac{1}{i}(\mathbf{r} \times \nabla).$$

87. The system of claim 85, wherein the coefficients $a_E(\ell,m)$ and $a_M(\ell,m)$ specify the amounts of electric (ℓ,m) multipole and magnetic (ℓ,m) multipole fields, and are determined by sources and boundary conditions as are the relative proportions in $g_\ell(kr)$.

88. The system of claim 86, wherein the electric and magnetic coefficients from the sources is

$$a_E(\ell,m) = \frac{4\pi k^2}{i\sqrt{\ell(\ell+1)}} \int Y_\ell^{m*} \left\{ \rho \frac{\delta}{\delta r}[r j_\ell(kr)] + \frac{ik}{c}(\mathbf{r} \cdot \mathbf{J}) j_\ell(kr) - ik\nabla \cdot (\mathbf{r} \times \mathbf{M}) j_\ell(kr) \right\} d^3x$$

and

$$a_M(\ell,m) = \frac{-4\pi k^2}{\sqrt{\ell(\ell+1)}} \int j_\ell(kr) Y_\ell^{m*} \mathbf{L} \cdot \left(\frac{\mathbf{J}}{c} + \nabla \times \mathbf{M} \right) d^3x$$

respectively, where the distribution of charge $\rho(\mathbf{x},t)$, current $\mathbf{J}(\mathbf{x},t)$, and intrinsic magnetization $M(\mathbf{x},t)$ are harmonically varying sources: $\rho(\mathbf{x})e^{-\omega_n t}$, $\mathbf{J}(\mathbf{x})e^{-\omega_n t}$, and $M(\mathbf{x})e^{-\omega_n t}$.

89. The system of claim 86, wherein the charge and intrinsic magnetization terms are zero.

90. The system of claim 87, wherein since the source dimensions are very small compared to a wavelength ($kr_{max} \ll 1$), the small argument limit can be used to give the magnetic multipole coefficient $a_M(\ell,m)$ as

$$a_M(\ell,m) = \frac{-4\pi k^{\ell+2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\left(M_{\ell m}+M'_{\ell m}\right) = \frac{-4\pi k^{\ell+2}}{\frac{(2\ell+1)!}{2^n n!}}\left(\frac{\ell+1}{\ell}\right)^{1/2}\left(M_{\ell m}+M'_{\ell m}\right)$$

5   where the magnetic multipole moments are

$$M_{\ell m} = -\frac{1}{\ell+1}\int r^\ell Y^*_{\ell m}\nabla\cdot\left(\frac{\mathbf{r}\times\mathbf{J}}{c}\right)d^3 x$$

$$M'_{\ell m} = -\int r^\ell Y^*_{\ell m}\nabla\cdot M d^3 x$$

91. The system of claim 88, wherein the geometrical factor of the surface current-density function of the orbitsphere about the z-axis is $\left(\dfrac{2}{3}\right)^{-1}$.

10

92. The system of claim 89, wherein the multipole coefficient $a_{Mag}(\ell,m)$ of the magnetic force is

$$a_{Mag}(\ell,m) = \frac{\frac{3}{2}}{(2\ell+1)!!}\frac{1}{\ell+2}\left(\frac{\ell+1}{\ell}\right)^{1/2}.$$

15   **93.** The system of claim 90, wherein for singlet states with $\ell \neq 0$, a minimum energy is achieved with conservation of the photon's angular momentum of $\hbar$ when the magnetic moments of the corresponding angular momenta relative to the electron velocity and corresponding Lorentzian forces superimpose negatively such that the spin component is radial (i_r-direction) and the orbital component is central ($-i_r$-

20   direction).

94. The system of claim 91, wherein the amplitude of the orbital angular momentum $\mathbf{L}_{rotational\ \text{orbital}}$, is

$$\mathbf{L} = I\omega i_z = \hbar\left[\frac{\ell(\ell+1)}{\ell^2+2\ell+1}\right]^{\frac{1}{2}} = \hbar\sqrt{\frac{\ell}{\ell+1}}.$$

25

95. The system of claim 92, wherein the magnetic force between the two electrons is

$$\mathbf{F}_{mag} = -\frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\frac{1}{\ell+2}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{2}\frac{\hbar^2}{m_e r^3}\left(\sqrt{s(s+1)}-\sqrt{\frac{\ell}{\ell+1}}\right).$$

96. The system of claim 93, wherein the force balance equation which achieves the condition that the sum of the mechanical momentum and electromagnetic momentum is

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} - \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r^3}\left(\sqrt{s(s+1)}-\sqrt{\frac{\ell}{\ell+1}}\right)$$

;

$$s = \frac{1}{2}.$$

97. The system of claim 94, wherein

$$r_2 = \left[n + \frac{\frac{3}{4}}{(2\ell+1)!!}\frac{1}{\ell+2}\left(\frac{\ell+1}{\ell}\right)^{1/2}\left(\sqrt{\frac{3}{4}}-\sqrt{\frac{\ell}{\ell+1}}\right)\right]a_{He} \qquad n = 2,3,4,...$$

98. The system of claim 95, wherein the excited-state energy is the energy stored in the electric field, E_{ele}, which is the energy of the excited-state electron (electron 2) relative to the ionized electron at rest having zero energy:

$$E_{ele} = -\frac{1}{n}\frac{e^2}{8\pi\varepsilon_o r_2}.$$

99. The system of claim 96, wherein using r_2, r_1 can be solved using the equal and opposite magnetic force of electron 2 on electron 1 and the central Coulombic force corresponding to the nuclear charge.

100. The system of claim 97, wherein the force balance between the centrifugal and electric and magnetic forces is

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} + \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r_2^3}\left(\sqrt{s(s+1)}-\sqrt{\frac{\ell}{\ell+1}}\right)$$

such that with $s = \dfrac{1}{2}$,

$$r_1^3 + \frac{n8r_1r_2^3}{3\left(\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right)}(2\ell+1)!!\left(\frac{\ell}{\ell+1}\right)^{1/2}(\ell+2)$$

$$-\frac{n4r_2^3}{3\left(\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right)}(2\ell+1)!!\left(\frac{\ell}{\ell+1}\right)^{1/2}(\ell+2) = 0 \qquad n = 2,3,4,\ldots$$

$$r_1 = r_{13} = \frac{1}{2} + \frac{1}{16g} + O\!\left(g^{-3/2}\right)$$

where r_1 and r_2 are in units of a_{He}.

5

101.   The system of claim 98, wherein for the triplet-excited state with $\ell \neq 0$, a
         minimum energy is achieved with conservation of the photon's angular momentum of
         $\hbar$ when the magnetic moments of the corresponding angular momenta superimpose
         negatively such that the spin component is central and the orbital component is

10   radial.

102.   The system of claim 99, wherein the spin is doubled such that the force balance
         equation is given by

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} + \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r^3}\left(2\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right);$$

15    $s = \dfrac{1}{2}$.

103.   The system of claim 100, wherein

$$r_2 = \left[n - \frac{\frac{3}{4}}{(2\ell+1)!!}\frac{1}{\ell+2}\left(\frac{\ell+1}{\ell}\right)^{1/2}\left(2\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right)\right]a_{He} \qquad n = 2,3,4,\ldots$$

20

104.   The system of claim 101, wherein the excited-state energy is the energy stored in
         the electric field, E_{ele}, which is the energy of the excited-state electron (electron 2)
         relative to the ionized electron at rest having zero energy:

$$E_{ele} = -\frac{1}{n}\frac{e^2}{8\pi\varepsilon_o r_2} \ .$$

105. The system of claim 102, wherein using r_2, r_1 can be solved using the equal and opposite magnetic force of electron 2 on electron 1 and the central Coulombic force corresponding to the nuclear charge.

106. The system of claim 103, wherein the force balance between the centrifugal and electric and magnetic forces is

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} - \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r_2^3}\left(2\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

such that with $s = \frac{1}{2}$,

$$r_1^3 - \frac{n8r_1 r_2^3}{3\left(\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right)}(2\ell+1)!!\left(\frac{\ell}{\ell+1}\right)^{1/2}(\ell+2)$$

$$+ \frac{n4r_2^3}{3\left(\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right)}(2\ell+1)!!\left(\frac{\ell}{\ell+1}\right)^{1/2}(\ell+2) = 0$$

$$n = 2,3,4,\ldots$$

$$r_1 = r_{13} = -\sqrt{\frac{2}{3}}g\left(\cos\frac{\theta}{3} - \sqrt{3}\sin\frac{\theta}{3}\right)$$

where r_1 and r_2 are in units of a_{He}.

107. The system of claim 104, wherein the spin-orbital coupling force is used in the force balance equation wherein the corresponding energy $E_{s/o}$ is given by

$$E_{s/o} = 2\frac{\alpha}{2\pi}\left(\frac{e\hbar}{2m_e}\right)\frac{\mu_0 e\hbar}{2(2\pi m_e)\left(\frac{r}{2\pi}\right)^3}\sqrt{\frac{3}{4}} = \frac{\alpha\pi\mu_0 e^2\hbar^2}{m_e^2 r^3}\sqrt{\frac{3}{4}} \ .$$

108. The system of claim 105, wherein the force balance also includes a term corresponding to the frequency shift derived after that of the Lamb shift.

109. The system of claim 106, wherein with $\Delta m_\ell = -1$ is included and the energy,

E_{FS}, for the $^{2}P_{3/2} \rightarrow {}^{2}P_{1/2}$ transition called the fine structure splitting is given by:

$$E_{FS} = \frac{\alpha^{5}(2\pi)^{2}}{8} m_{e}c^{2} \sqrt{\frac{3}{4}} + \left(13.5983\ eV\left(1 - \frac{1}{2^{2}}\right)\right)^{2} \left[\frac{\left(\frac{3}{4\pi}\left(1 - \sqrt{\frac{3}{4}}\right)\right)^{2}}{2h\mu_{e}c^{2}} + \frac{\left(1 + \left(1 - \sqrt{\frac{3}{4}}\right)\right)^{2}}{2hm_{H}c^{2}} \right]$$

$$= 4.5190\ X\ 10^{-5}\ eV + 1.75407\ X\ 10^{-7}\ eV$$

$$= 4.53659\ X\ 10^{-5}\ eV$$

where the first term corresponds to $E_{s/o}$ expressed in terms of the mass energy of the electron and the second and third terms correspond to the electron recoil and atom recoil,

5   respectively.

110.   A method comprising the steps of;

a.) inputting the electron functions that obey Maxwell's equations;

b.) determining the corresponding centrifugal, Coulombic, diamagnetic and

10   paramagnetic forces for a given set of quantum numbers corresponding to a solution of

Maxwell's equations for at least one photon and one electron of the excited state;

d.) forming the force balance equation comprising the centrifugal force equal to

the sum of the Coulombic, diamagnetic and paramagnetic forces;

e.) solving the force balance equation for the electron radii;

15   f.) calculating the energy of the electrons using the radii and the corresponding

electric and magnetic energies, and

g.) outputting the calculated energy of the electons.

111.   The method according to claim 110, further comprising displaying the electron

20   function at each radius to visualize the excited-state atom or atomic ion.

112.   The method according to claim 110, wherein the output is rendered using the

electron functions.

25   113.   The method according to claim 110, wherein the electron functions are given by

at least one of the group comprising:

$\ell = 0$

$$\rho(r,\theta,\phi,t) = \frac{e}{8\pi r^2}[\delta(r-r_n)]\left[Y_0^0(\theta,\phi) + Y_\ell^m(\theta,\phi)\right]$$

$\ell \neq 0$

5

$$\rho(r,\theta,\phi,t) = \frac{e}{4\pi r^2}[\delta(r-r_n)]\left[Y_0^0(\theta,\phi) + \text{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\}\right]$$

where $Y_\ell^m(\theta,\phi)$ are the spherical harmonic functions that spin about the z-axis with angular

frequency ω_n with $Y_0^0(\theta,\phi)$ the constant function.

10    $\text{Re}\left\{Y_\ell^m(\theta,\phi)e^{i\omega_n t}\right\} = P_\ell^m(\cos\theta)\cos(m\phi + \omega_n' t)$ where to keep the form of the spherical harmonic

as a traveling wave about the z-axis, $\omega_n' = m\omega_n$.

114.   The method according to claim 113, wherein the force balance equation is given

by at least one of the group comprising:

15    $$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} + \frac{2}{3}\frac{1}{n}\frac{\hbar^2}{2m_e r_2^3}\sqrt{s(s+1)}$$

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} - \frac{1}{3n}\frac{\hbar^2}{m_e r_2^3}\sqrt{s(s+1)}$$

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} + 2\frac{2}{3}\frac{1}{n}\frac{\hbar^2}{2m_e r_2^3}\sqrt{s(s+1)}$$

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} - \frac{2}{3n}\frac{\hbar^2}{m_e r_2^3}\sqrt{s(s+1)}$$

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} - \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r^3}\left(\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

20    $$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} + \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r_2^3}\left(\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} + \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r^3}\left(2\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} - \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r_2^3}\left(2\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

wherein $s = \frac{1}{2}$.

115.   The method according to claim 113, wherein the force is given by at least one of

the forces of the force balance equations of the group comprising:

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} + \frac{2}{3}\frac{1}{n}\frac{\hbar^2}{2m_e r_2^3}\sqrt{s(s+1)}$$

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} - \frac{1}{3n}\frac{\hbar^2}{m_e r_2^3}\sqrt{s(s+1)}$$

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} + 2\frac{2}{3}\frac{1}{n}\frac{\hbar^2}{2m_e r_2^3}\sqrt{s(s+1)}$$

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} - \frac{2}{3n}\frac{\hbar^2}{m_e r_2^3}\sqrt{s(s+1)}$$

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} - \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r^3}\left(\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} + \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r_2^3}\left(\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

$$\frac{m_e v^2}{r_2} = \frac{\hbar^2}{m_e r_2^3} = \frac{1}{n}\frac{e^2}{4\pi\varepsilon_o r_2^2} + \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r^3}\left(2\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

$$\frac{m_e v^2}{r_1} = \frac{\hbar^2}{m_e r_1^3} = \frac{2e^2}{4\pi\varepsilon_o r_1^2} - \frac{1}{n}\frac{\frac{3}{2}}{(2\ell+1)!!}\left(\frac{\ell+1}{\ell}\right)^{1/2}\frac{1}{\ell+2}\frac{1}{2}\frac{\hbar^2}{m_e r_2^3}\left(2\sqrt{s(s+1)} - \sqrt{\frac{\ell}{\ell+1}}\right)$$

wherein $s = \frac{1}{2}$.

116.   The method according to claim 110, wherein the radii are given by at least one

of the group comprising:

$$r_2 = \left[n - \frac{\sqrt{\frac{3}{4}}}{3} \right] a_{He} \qquad n = 2, 3, 4, \ldots$$

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$$r_1 = r_{13} = -\sqrt{\frac{2}{3}}\, g\left(\cos\frac{\theta}{3} - \sqrt{3}\sin\frac{\theta}{3}\right)$$

$$r_2 = \left[n - \frac{2\sqrt{\frac{3}{4}}}{3} \right] a_{He} \qquad n = 2,3,4,\dots$$

$$r_1 = r_{13} = -\sqrt{\frac{2}{3}}\, g\left(\cos\frac{\theta}{3} - \sqrt{3}\sin\frac{\theta}{3}\right)$$

$$r_2 = \left[n + \frac{\frac{3}{4}}{(2\ell+1)!!}\frac{1}{\ell+2}\left(\frac{\ell+1}{\ell}\right)^{1/2}\left(\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right) \right] a_{He} \qquad n = 2,3,4,\dots$$

5

$$r_1 = r_{11} = \sqrt[3]{-\frac{g}{2}}\left\{ \sqrt[3]{1 + \sqrt{1 - \frac{32}{27}g}} - \sqrt[3]{\sqrt{1 - \frac{32}{27}g} - 1} \right\}$$

$$r_2 = \left[n - \frac{\frac{3}{4}}{(2\ell+1)!!}\frac{1}{\ell+2}\left(\frac{\ell+1}{\ell}\right)^{1/2}\left(2\sqrt{\frac{3}{4}} - \sqrt{\frac{\ell}{\ell+1}}\right) \right] a_{He} \qquad n = 2,3,4,\dots$$

$$r_1 = r_{13} = -\sqrt{\frac{2}{3}}\, g\left(\cos\frac{\theta}{3} - \sqrt{3}\sin\frac{\theta}{3}\right)$$

where r_1 and r_2 are in units of a_{He}.

10   **117.**   The method according to claim 110, wherein the electric energy of each electron of radius r_n is given by:

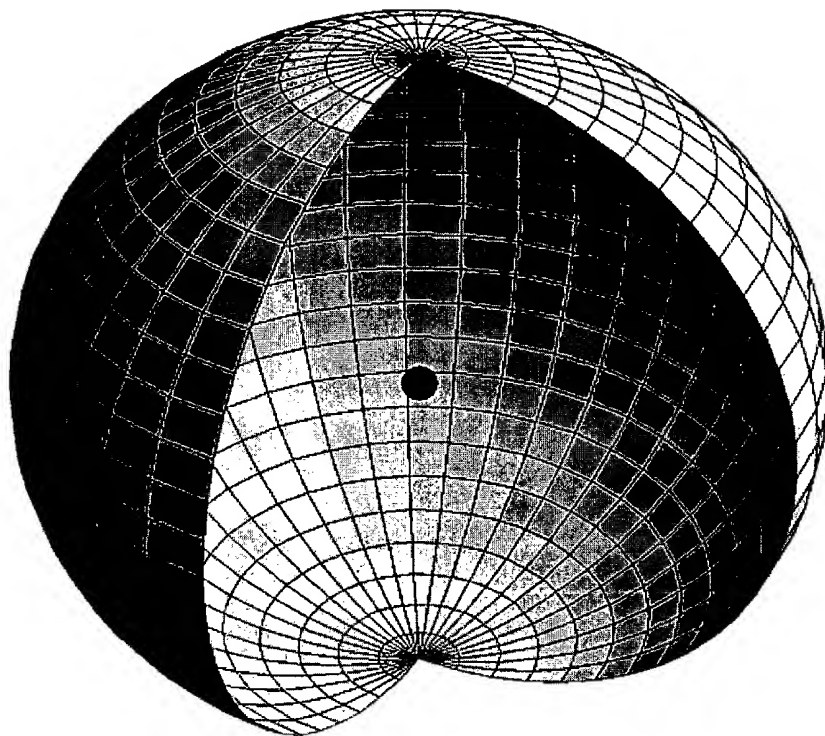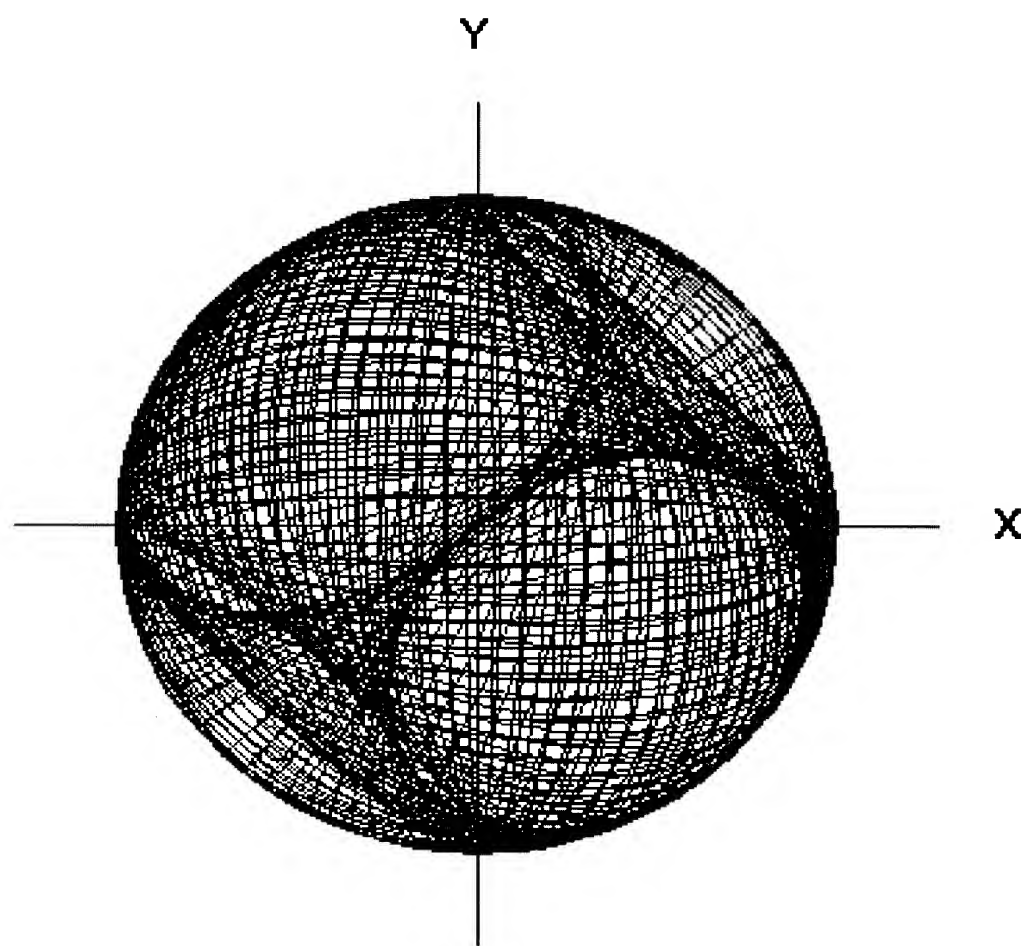$$E_{ele} = -\frac{1}{n}\frac{e^2}{8\pi\varepsilon_o r_2}$$

Fig. 1

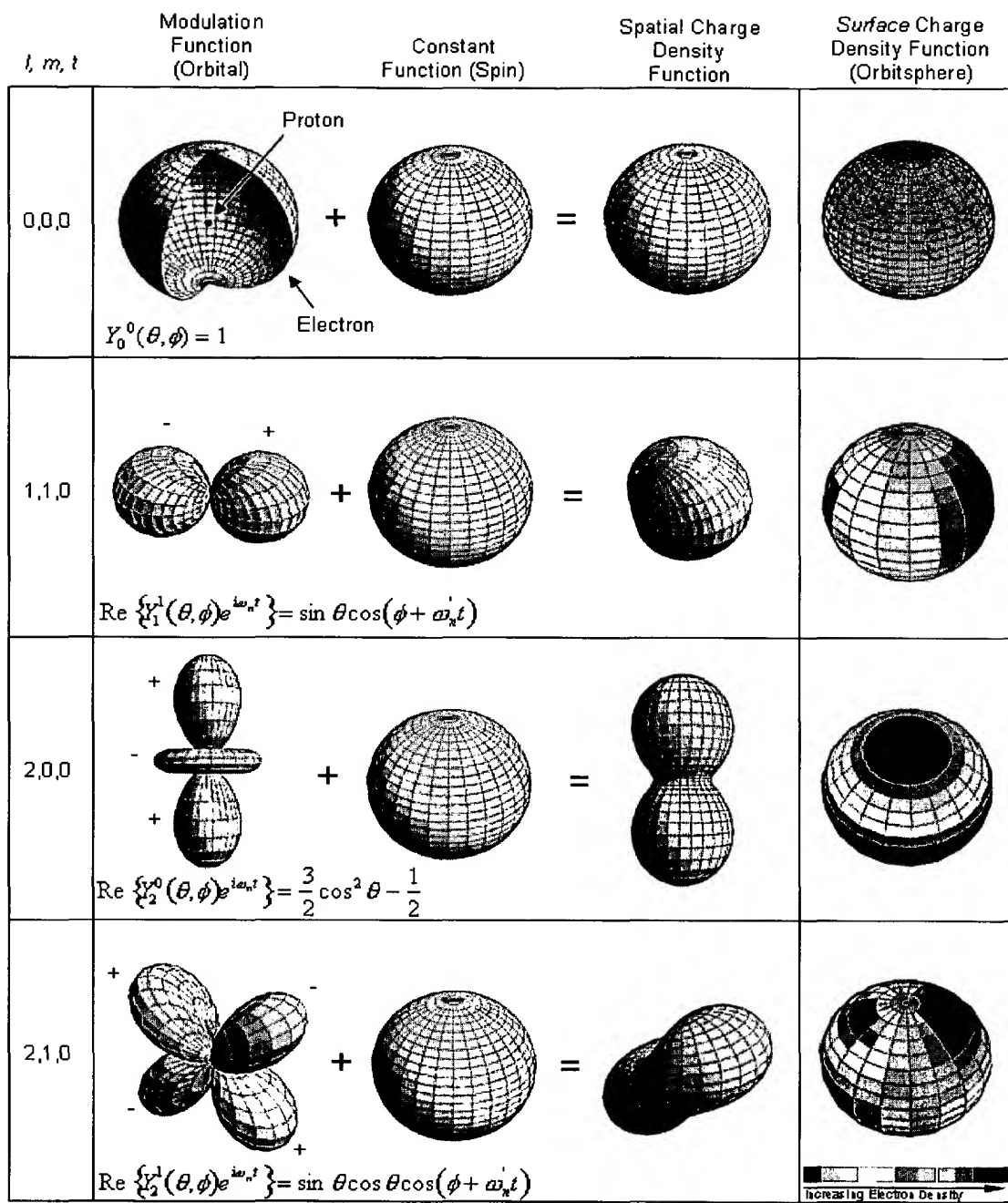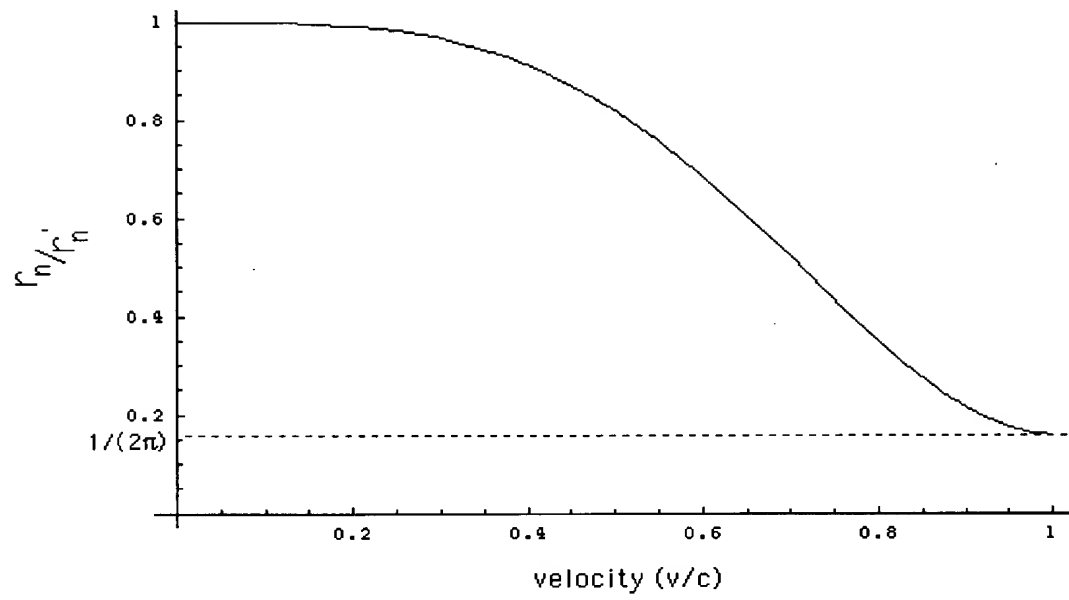Fig. 2

View Along the Positive Z Axis

Fig. 3
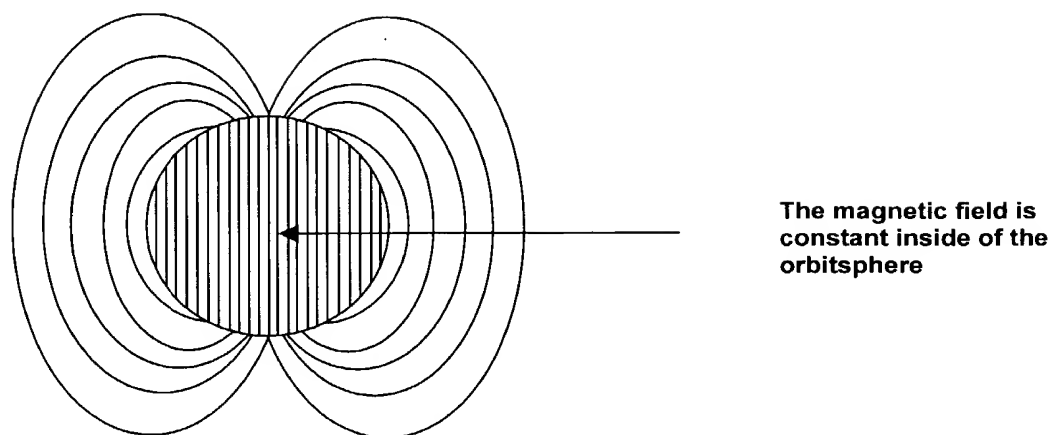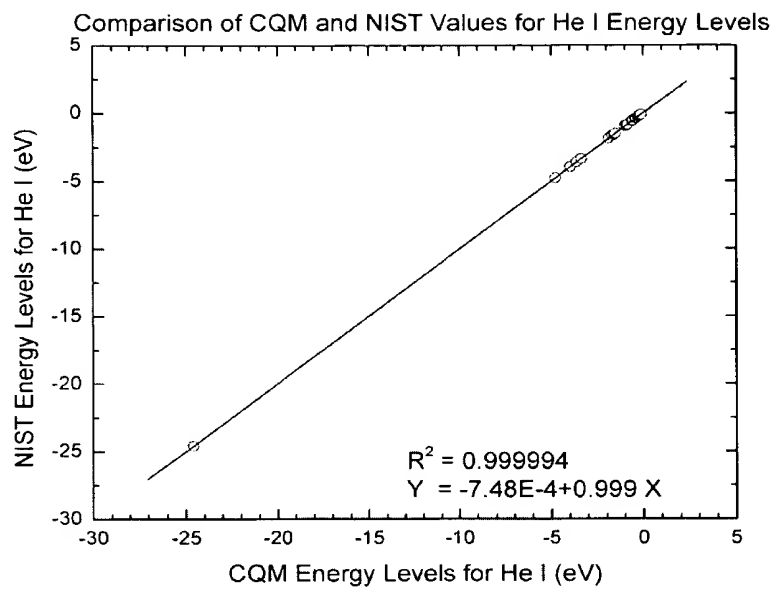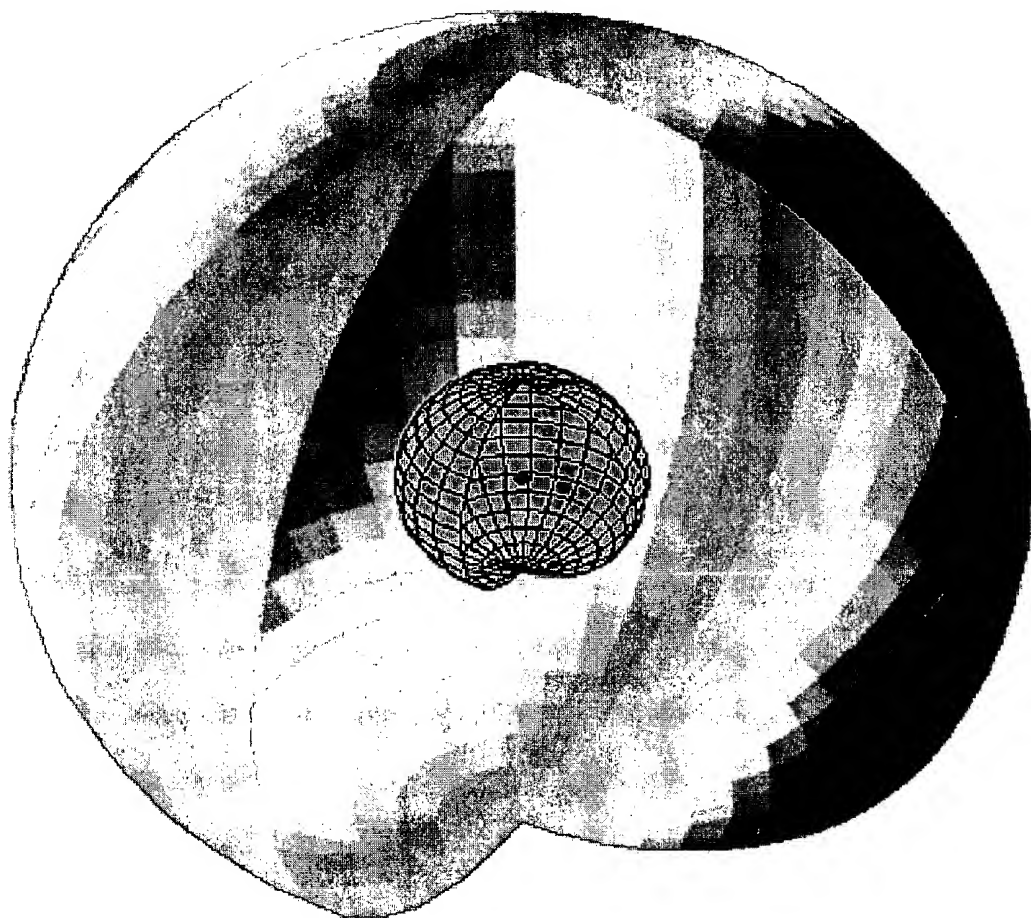
l, m, t	Modulation Function (Orbital)		Constant Function (Spin)		Spatial Charge Density Function	*Surface* Charge Density Function (Orbitsphere)
0,0,0	$Y_0^0(\theta,\phi) = 1$	+		=		
1,1,0	$\mathrm{Re}\left\{Y_1^1(\theta,\phi)e^{i\omega_n t}\right\} = \sin\theta\cos(\phi + \omega_n' t)$	+		=		
2,0,0	$\mathrm{Re}\left\{Y_2^0(\theta,\phi)e^{i\omega_n t}\right\} = \frac{3}{2}\cos^2\theta - \frac{1}{2}$	+		=		
2,1,0	$\mathrm{Re}\left\{Y_2^1(\theta,\phi)e^{i\omega_n t}\right\} = \sin\theta\cos\theta\cos(\phi + \omega_n' t)$	+		=		Increasing Electron Density

Fig. 4

Fig. 5

The magnetic field is constant inside of the orbitsphere

Fig. 5

Fig. 6

Comparison of CQM and NIST Values for He I Energy Levels

$R^2 = 0.999994$
$Y = -7.48E\text{-}4 + 0.999\ X$

Fig. 7

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(71) Applicant *(for all designated States except US)*: **BLACK-LIGHT POWER, INC.** [US/US]; 493 Old Trenton Road, Cranbury, NJ 08512 (US).

(72) Inventor; and
(75) Inventor/Applicant *(for US only)*: **MILLS, Randell, L.** [US/US]; 27 Rosedale Road, Princeton, NJ 08540 (US).

(74) Agent: **MELCHER, Jeffrey, S.**; Manelli, Denison & Selter, PLLC, 2000 M Street, N.W., 7th Floor, Washington, DC 20036-3307 (US).

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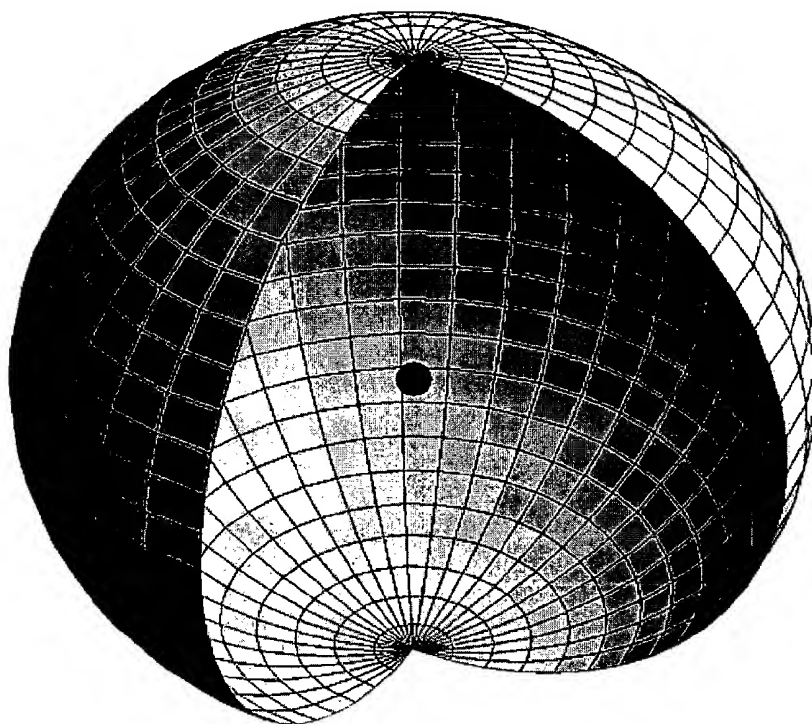
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(54) Title: METHOD AND SYSTEM OF COMPUTING AND RENDERING THE NATURE OF THE EXCITED ELECTRONIC STATES OF ATOMS AND ATOMIC IONS

(57) Abstract: A method and system of physically solving the charge, mass, and current density functions of excited-state atoms and atomic ions using Maxwell's equations and computing and rendering the nature of excited-state electrons using the solutions. The results can be displayed on visual or graphical media. The display can be static or dynamic such that electron spin and rotation motion can be displayed in an embodiment. The displayed information is useful to anticipate reactivity and physical properties. The insight into the nature of excited-state electrons can permit the solution and display of those of other atoms and atomic ions and provide utility to anticipate their reactivity and physical properties as well as spectral absorption and emission to lead to new optical materials and light sources.

INTERNATIONAL SEARCH REPORT

A. CLASSIFICATION OF SUBJECT MATTER

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B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
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Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)
East Search

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category *	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	JP 2002008892 (Shimazu) 11 January 2002 (11.01.2002), Solution, lines 1-6 and Figs. 6-8.	1-16
A	US 6,024,935 (Mills et al) 15 February 2000 (15.02.2000).	1-117

☐ Further documents are listed in the continuation of Box C.   ☐ See patent family annex.

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Facsimile No. (571) 273-3201	

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